1

(解説)

(1) $xy + xz = x \times y + x \times z$
$= x(y + z)$

(2) $3ax - 2ay + az = a \times 3x - a \times 2y + a \times z$
$= a(3x - 2y + z)$

(3) $4x - 6y = 2 \times 2x - 2 \times 3y$
$= 2(2x - 3y)$

(4) $-2xy + 6yz = -2y \times x - 2y \times (-3z)$
$= -2y(x - 3z)$

(5) $abc - bcd = bc \times a - bc \times d$
$= bc(a - d)$

(6) $\dfrac{2}{3}ab + \dfrac{1}{2}ac = \dfrac{4}{6}ab + \dfrac{3}{6}ac$
$= \dfrac{1}{6}a \times 4b + \dfrac{1}{6}a \times 3c$
$= \dfrac{1}{6}a(4b + 3c)$

(7) $xy + x = x \times y + x \times 1$
$= x(y + 1)$

(8) $2mx^2 + 3m^2x = mx \times 2x + mx \times 3m$
$= mx(2x + 3m)$

(9) $12x^3y^2 - 8x^2y = 4x^2y \times 3xy - 4x^2y \times 2$
$= 4x^2y(3xy - 2)$

(10) $4a^2bc + 8ab^2c - 6abc^2 = 2abc \times 2a + 2abc \times 4b - 2abc \times 3c$
$= 2abc(2a + 4b - 3c)$

(11) $(a + b)x - (a + b)y = (a + b)(x - y)$

(12) $(a - 3b)x - (3b - a)y = (a - 3b)x + (a - 3b)y$
$= (a - 3b)(x + y)$

2

(解説)

(1) $x^2 + 7x + 10 = x^2 + (2 + 5)x + 2 \times 5$
$= (x + 2)(x + 5)$

(2) $a^2 - 3a - 28 = a^2 + \{(-7) + 4\}a + (-7) \times 4$
$= (a - 7)(a + 4)$

(3) $x^2 + 9x - 36 = x^2 + \{12 + (-3)\}x + 12 \times (-3)$
$= (x + 12)(x - 3)$

(4) $x^2 + 4xy - 32y^2 = x^2 + (-4y + 8y)x + (-4y) \cdot 8y$
$= (x - 4y)(x + 8y)$

3

(解説)

(1) $x^2 + 12x + 36 = x^2 + 2 \times 6 \times x + 6^2$
$= (x + 6)^2$

(2) $a^2 - 18a + 81 = a^2 - 2 \times 9 \times a + 9^2$
$= (a - 9)^2$

(3) $4x^2 + 4x + 1 = (2x)^2 + 2 \times 1 \times 2x + 1^2$
$= (2x + 1)^2$

(4) $25x^2 - 70xy + 49y^2 = (5x)^2 - 2 \times 7y \times 5x + (7y)^2$
$= (5x - 7y)^2$

4

(解説)

(1) $x^2 - 36 = x^2 - 6^2$
$= (x + 6)(x - 6)$

(2) $25x^2 - 64a^2 = (5x)^2 - (8a)^2$
$= (5x + 8a)(5x - 8a)$

5

(解説)

(1) $x^2 - 5x - 14 = (x + 2)(x - 7)$

(2) $x^2 - 14x + 49 = (x - 7)^2$

(3) $x^2 - 81 = (x + 9)(x - 9)$

6

(解説)

(1) $(2x - 1)^2 - 3(x + 1)(x - 1) = (4x^2 - 4x + 1) - 3(x^2 - 1)$
$= 4x^2 - 4x + 1 - 3x^2 + 3$
$= x^2 - 4x + 4$
$= (x - 2)^2$

(2) $(x + 1)(x - 4) - (5 - 3x) = (x^2 - 3x - 4) - 5 + 3x$
$= x^2 - 9$
$= (x + 3)(x - 3)$

(3) $5(x + 2)(x - 2) - (2x + 1)^2 = 5(x^2 - 4) - (4x^2 + 4x + 1)$
$= 5x^2 - 20 - 4x^2 - 4x - 1$
$= x^2 - 4x - 21$
$= (x + 3)(x - 7)$

7

(解説)

(1) $2x^2 + 5x + 3 = (x + 1)(2x + 3)$

$$
\begin{array}{ccc}
1 & \diagtimes & 1 \longrightarrow 2 \\
2 & & 3 \longrightarrow 3 \\
\hline
2 & 3 & 5
\end{array}
$$

(2) $2x^2 - 7x + 3 = (x - 3)(2x - 1)$

$$
\begin{array}{ccc}
1 & \diagtimes & -3 \longrightarrow -6 \\
2 & & -1 \longrightarrow -1 \\
\hline
2 & 3 & -7
\end{array}
$$

(3) $3x^2 + 7x - 6 = (x + 3)(3x - 2)$

$$
\begin{array}{ccc}
1 & \diagtimes & 3 \longrightarrow 9 \\
3 & & -2 \longrightarrow -2 \\
\hline
3 & -6 & 7
\end{array}
$$

(4) $6x^2 - 7xy - 20y^2 = (2x - 5y)(3x + 4y)$

$$
\begin{array}{ccc}
2 & \diagtimes & -5 \longrightarrow -15 \\
3 & & 4 \longrightarrow 8 \\
\hline
6 & -20 & -7
\end{array}
$$

8

(解説)

(1) $-4x^2 + 20x - 24 = -4(x^2 - 5x + 6)$
$= -4(x - 2)(x - 3)$

(2) $2ax^2 - 4ax - 30a = 2a(x^2 - 2x - 15)$
$= 2a(x + 3)(x - 5)$

(3) $2x^2y - 16xy + 32y = 2y(x^2 - 8x + 16)$
$= 2y(x - 4)^2$

9

(解説)

$x^4 - 81 = (x^2)^2 - 9^2$ 　　　　$\}$ $x^2 - a^2$ の因数分解
$= (x^2 + 9)(x^2 - 9)$
$= (x^2 + 9)(x^2 - 3^2)$ 　$\}$ $x^2 - 9$ はさらに 因数分解できる
$= (x^2 + 9)(x + 3)(x - 3)$　答

10

(解説)

$x^4 - 13x^2 + 36 = (x^2)^2 - 13x^2 + 36 = (x^2 - 4)(x^2 - 9) = (x + 2)(x - 2)(x + 3)(x - 3)$

11

(解説)

(1) $a + b = A$ とおくと
$(a + b)^2 - 6(a + b) + 5 = A^2 - 6A + 5 = (A - 1)(A - 5) = (a + b - 1)(a + b - 5)$

(2) $x + 2 = A$ とおくと
$(x + 2)^2 - 6(x + 2) - 16 = A^2 - 6A - 16 = (A + 2)(A - 8)$
$= \{(x + 2) + 2\}\{(x + 2) - 8\} = (x + 4)(x - 6)$

12

(解説)

$(x^2 - 2x)^2 - 11(x^2 - 2x) + 24 = \{(x^2 - 2x) - 3\}\{(x^2 - 2x) - 8\} = (x^2 - 2x - 3)(x^2 - 2x - 8)$
$= (x + 1)(x - 3)(x + 2)(x - 4) = (x + 1)(x + 2)(x - 3)(x - 4)$

13

(解説)

$(x + 1)(x + 3)(x + 4)(x + 6) + 8 = (x + 1)(x + 6) \times (x + 3)(x + 4) + 8$
$= (x^2 + 7x + 6)(x^2 + 7x + 12) + 8$
$= (x^2 + 7x)^2 + 18(x^2 + 7x) + 80 = (x^2 + 7x + 10)(x^2 + 7x + 8)$
$= (x + 2)(x + 5)(x^2 + 7x + 8)$

14

(解説)

(1) $ac+ad+bc+bd$
$=a(c+d)+(bc+bd)$　　　$\leftarrow c+d=M$ とおくと
$=a(c+d)+b(c+d)$　　　$aM+bM=(a+b)M$
$=(a+b)(c+d)$

(2) $3ax-bx+15ay-5by=x(3a-b)+5y(3a-b)$
$=(3a-b)(x+5y)$

(3) $ab+bc-cd-da=b(a+c)-d(c+a)$
$=(a+c)(b-d)$

(4) $ab-7a-b+7=a(b-7)-(b-7)$
$=(a-1)(b-7)$

(5) $xy-yz+zu-ux=y(x-z)+u(z-x)$
$=y(x-z)-u(x-z)$
$=(x-z)(y-u)$

15

(解説)

(1) $x^2-6x+9-4y^2=(x^2-6x+9)-(2y)^2$
$=(x-3)^2-(2y)^2$
$=\{(x-3)+2y\}\{(x-3)-2y\}$
$=(x+2y-3)(x-2y-3)$

(2) $9a^2-16b^2+40b-25=(3a)^2-(16b^2-40b+25)$
$=(3a)^2-(4b-5)^2$
$=\{3a+(4b-5)\}\{3a-(4b-5)\}$
$=(3a+4b-5)(3a-4b+5)$

1

(解説)

(1) $xy+2y=y\times x+y\times 2$
$=y(x+2)$

(2) $ab-bc=b\times a-b\times c$
$=b(a-c)$

(3) $b-ab=b\times 1-b\times a$
$=b(1-a)$

(4) $3ax-2ay+4az=a\times 3x-a\times 2y+a\times 4z=a(3x-2y+4z)$

(5) $6x^4+12x^2=6x^2(x^2+2)$

(6) $m^2ab-ma^2b=mab(m-a)$

(7) $-4a^2bc-6ab^2c+2ac^2=-2ac(2ab+3b^2-c)$

(8) $5x^2y^3z-15xy^2z^3-10x^3yz^2=5xyz(xy^2-3yz^2-2x^2z)$

(9) $\frac{1}{2}m^2n-\frac{3}{2}mn^2=\frac{1}{2}mn\times m-\frac{1}{2}mn\times 3n=\frac{1}{2}mn(m-3n)$

(10) $3x^3-x^2+\frac{1}{6}x=\frac{x}{6}\times 18x^2-\frac{x}{6}\times 6x+\frac{x}{6}\times 1=\frac{x}{6}(18x^2-6x+1)$

(11) $a(x-y)-9(x-y)=(x-y)(a-9)$

(12) $a(5a-3b)+b(3b-5a)=a(5a-3b)-b(5a-3b)=(a-b)(5a-3b)$

(13) $(a-b)^2+c(b-a)=(a-b)^2-c(a-b)$
$=(a-b)\{(a-b)-c\}$
$=(a-b)(a-b-c)$

2

(解説)

(1) $x^2+7x+10=(x+2)(x+5)$　　　(2) $x^2+12x+35=(x+5)(x+7)$

(3) $x^2-5x+4=(x-1)(x-4)$　　　(4) $x^2-10x+21=(x-3)(x-7)$

(5) $x^2-2x-3=(x+1)(x-3)$　　　(6) $x^2+4x-12=(x-2)(x+6)$

(7) $x^2+5x-24=(x-3)(x+8)$　　　(8) $x^2-4x-21=(x+3)(x-7)$

(9) $x^2+17x+30=(x+2)(x+15)$　　(10) $x^2+5x-50=(x-5)(x+10)$

(11) $x^2-16x+55=(x-5)(x-11)$　　(12) $x^2+10x-56=(x-4)(x+14)$

(13) $x^2+12x-108=(x-6)(x+18)$　　(14) $a^2-33a+90=(a-3)(a-30)$

(15) $y^2+4y-96=(y-8)(y+12)$　　(16) $z^2+15z+36=(z+3)(z+12)$

(17) $t^2+18t+77=(t+7)(t+11)$　　(18) $p^2-13p-30=(p+2)(p-15)$

(19) $x^2+14xy+48y^2=(x+6y)(x+8y)$　　(20) $x^2-9xy-36y^2=(x+3y)(x-12y)$

(21) $p^2+2pq-35q^2=(p+7q)(p-5q)$　　(22) $a^2-16ab+48b^2=(a-4b)(a-12b)$

(23) $x^2-xy-12y^2=(x+3y)(x-4y)$　　(24) $a^2+5ab-150b^2=(a+15b)(a-10b)$

3

(解説)

(1) $x^2+6x+9=(x+3)^2$　　　(2) $x^2+8x+16=(x+4)^2$

(3) $x^2-14x+49=(x-7)^2$　　　(4) $x^2-20x+100=(x-10)^2$

(5) $9x^2+30x+25=(3x+5)^2$　　(6) $16a^2-24a+9=(4a-3)^2$

(7) $49t^2-28t+4=(7t-2)^2$　　(8) $25p^2+80p+64=(5p+8)^2$

(9) $x^2+10xy+25y^2=(x+5y)^2$　　(10) $81a^2+72ab+16b^2=(9a+4b)^2$

(11) $4a^2-12ab+9b^2=(2a-3b)^2$　　(12) $9x^2-48xy+64y^2=(3x-8y)^2$

4

(解説)

(1) $x^2-4=(x+2)(x-2)$

(2) $x^2-64=(x+8)(x-8)$

(3) $a^2-100=(a+10)(a-10)$

(4) $t^2-16=(t+4)(t-4)$

(5) $x^2-81y^2=(x+9y)(x-9y)$

(6) $a^2-49b^2=(a+7b)(a-7b)$

(7) $25x^2-36=(5x+6)(5x-6)$

(8) $64p^2-121=(8p+11)(8p-11)$

(9) $4y^2-81=(2y+9)(2y-9)$

(10) $9x^2-y^2=(3x+y)(3x-y)$

(11) $81a^2-169b^2=(9a+13b)(9a-13b)$

(12) $144x^2-25y^2=(12x+5y)(12x-5y)$

5

(解説)

(1) $x^2+7x+12=(x+3)(x+4)$　　　(2) $x^2+6x+9=(x+3)^2$

(3) $x^2-81=(x+9)(x-9)$　　　(4) $x^2-12x+35=(x-5)(x-7)$

(5) $x^2-8x+16=(x-4)^2$　　　(6) $x^2-49=(x+7)(x-7)$

(7) $x^2-10x+25=(x-5)^2$　　　(8) $9x^2-16=(3x+4)(3x-4)$

(9) $x^2-3x-18=(x+3)(x-6)$　　(10) $x^2-x-42=(x+6)(x-7)$

(11) $64a^2-b^2=(8a+b)(8a-b)$　　(12) $a^2-10ab+25b^2=(a-5b)^2$

(13) $a^2+7a-18=(a-2)(a+9)$　　(14) $a^2+14a+24=(a+2)(a+12)$

(15) $9x^2-64y^2=(3x+8y)(3x-8y)$　　(16) $x^2-5ax-36a^2=(x+4a)(x-9a)$

(17) $9a^2-42ab+49b^2=(3a-7b)^2$　　(18) $y^2-13y+30=(y-3)(y-10)$

(19) $4x^2+4xy+y^2=(2x+y)^2$　　(20) $25a^2-81=(5a+9)(5a-9)$

(21) $x^2-11x+30=(x-5)(x-6)$　　(22) $x^2+4xy-12y^2=(x+6y)(x-2y)$

(23) $a^2+12ab+36b^2=(a+6b)^2$　　(24) $25x^2+40xy+16y^2=(5x+4y)^2$

(25) $4x^2-25y^2=(2x+5y)(2x-5y)$　　(26) $y^2+7y-44=(y-4)(y+11)$

(27) $36x^2y^2-49=(6xy+7)(6xy-7)$　　(28) $16x^2+24xy+9y^2=(4x+3y)^2$

(29) $a^2-13ab+42b^2=(a-6b)(a-7b)$　　(30) $9a^2-12ab+4b^2=(3a-2b)^2$

6

(解説)

(1) $(2x+1)^2 - 3x(x+5) + 23 = 4x^2 + 4x + 1 - 3x^2 - 15x + 23$
$\qquad\qquad = x^2 - 11x + 24$
$\qquad\qquad = (x-3)(x-8)$

(2) $(2x+1)(3x+2) - (x+1)(5x+3) + x - 8$
$= 6x^2 + 4x + 3x + 2 - (5x^2 + 3x + 5x + 3) + x - 8$
$= x^2 - 9$
$= (x+3)(x-3)$

(3) $(3x+4)(4x+3) - (3x+2)^2 - 2x^2 + x + 37$
$= (12x^2 + 25x + 12) - (9x^2 + 12x + 4) - 2x^2 + x + 37$
$= x^2 + 14x + 45$
$= (x+5)(x+9)$

(4) $(x+3y)(x-3y) + 2y(x+5y) = x^2 - 9y^2 + 2xy + 10y^2$
$\qquad\qquad = x^2 + 2xy + y^2$
$\qquad\qquad = (x+y)^2$

(5) $2a(a+2b) - 2b(2b+3a) - (a+b)(a-b)$
$= 2a^2 + 4ab - 4b^2 - 6ab - (a^2 - b^2)$
$= a^2 - 2ab - 3b^2$
$= (a+b)(a-3b)$

7

(解説)

(1) $3x^2 + 5x + 2 = (x+1)(3x+2)$

1	$\times$	1	$\longrightarrow$	3
3		2	$\longrightarrow$	2
3		2		5

(2) $2x^2 + 7x + 3 = (x+3)(2x+1)$

1	$\times$	3	$\longrightarrow$	6
2		1	$\longrightarrow$	1
2		3		7

(3) $3x^2 - 7x + 2 = (x-2)(3x-1)$

1	$\times$	-2	$\longrightarrow$	-6
3		-1	$\longrightarrow$	-1
3		2		-7

(4) $6x^2 + x - 1 = (2x+1)(3x-1)$

2	$\times$	1	$\longrightarrow$	3
3		-1	$\longrightarrow$	-2
6		-1		1

(5) $8y^2 + 14y - 15 = (2y+5)(4y-3)$

2	$\times$	5	$\longrightarrow$	20
4		-3	$\longrightarrow$	-6
8		-15		14

(6) $6y^2 - 5y - 4 = (2y+1)(3y-4)$

2	$\times$	1	$\longrightarrow$	3
3		-4	$\longrightarrow$	-8
6		-4		-5

(7) $2x^2 - 7ax + 6a^2 = (x-2a)(2x-3a)$

1	$\times$	$-2a$	$\longrightarrow$	$-4a$
2		$-3a$	$\longrightarrow$	$-3a$
2		$6a^2$		$-7a$

(8) $3x^2 - 4ax - 4a^2 = (x-2a)(3x+2a)$

1	$\times$	$-2a$	$\longrightarrow$	$-6a$
3		$2a$	$\longrightarrow$	$2a$
3		$-4a^2$		$-4a$

(9) $5x^2 + 7xy - 6y^2 = (x+2y)(5x-3y)$

1	$\times$	$2y$	$\longrightarrow$	$10y$
5		$-3y$	$\longrightarrow$	$-3y$
5		$-6y^2$		$7y$

(10) $12x^2 - 7xy - 12y^2 = (3x-4y)(4x+3y)$

3	$\times$	$-4y$	$\longrightarrow$	$-16y$
4		$3y$	$\longrightarrow$	$9y$
12		$-12y^2$		$-7y$

(11) $6a^2 + 17ab + 12b^2 = (2a+3b)(3a+4b)$

2	$\times$	$3b$	$\longrightarrow$	$9b$
3		$4b$	$\longrightarrow$	$8b$
6		$12b^2$		$17b$

(12) $12a^2 - 23ab + 10b^2 = (3a-2b)(4a-5b)$

3	$\times$	$-2b$	$\longrightarrow$	$-8b$
4		$-5b$	$\longrightarrow$	$-15b$
12		$10b^2$		$-23b$

8

(解説)

(1) $3x^2 - 18x + 24 = 3(x^2 - 6x + 8) = 3(x-2)(x-4)$

(2) $5x^2 + 30x - 200 = 5(x^2 + 6x - 40) = 5(x-4)(x+10)$

(3) $-4x^2 + 8x + 12 = -4(x^2 - 2x - 3) = -4(x+1)(x-3)$

(4) $\dfrac{9}{7}x^2 + \dfrac{30}{7}x + \dfrac{25}{7} = \dfrac{1}{7}(9x^2 + 30x + 25) = \dfrac{1}{7}(3x+5)^2$

(5) $72x^2 - 242y^2 = 2(36x^2 - 121y^2) = 2(6x+11y)(6x-11y)$

(6) $\dfrac{144}{11}a^2 - \dfrac{25}{11}b^2 = \dfrac{1}{11}(144a^2 - 25b^2) = \dfrac{1}{11}(12a+5b)(12a-5b)$

(7) $5ax^2 + 25ax + 20a = 5a(x^2 + 5x + 4) = 5a(x+1)(x+4)$

(8) $3tx^2 - 15tx + 18t = 3t(x^2 - 5x + 6) = 3t(x-2)(x-3)$

(9) $-2mx^2 - 4mx + 30m = -2m(x^2 + 2x - 15) = -2m(x-3)(x+5)$

(10) $6px^2 - 96p = 6p(x^2 - 16) = 6p(x+4)(x-4)$

(11) $\dfrac{1}{7}ax^2 - \dfrac{13}{7}ax + \dfrac{40}{7}a = \dfrac{1}{7}a(x^2 - 13x + 40) = \dfrac{1}{7}a(x-5)(x-8)$

(12) $\dfrac{1}{7}a^2t + \dfrac{3}{7}abt - \dfrac{10}{7}b^2t = \dfrac{1}{7}t(a^2 + 3ab - 10b^2) = \dfrac{1}{7}t(a-2b)(a+5b)$

(13) $3ax^2 - 24ax + 36a = 3a(x^2 - 8x + 12) = 3a(x-2)(x-6)$

(14) $\dfrac{1}{2}a^2x - \dfrac{9}{2}b^2x = \dfrac{1}{2}x(a^2 - 9b^2) = \dfrac{1}{2}x(a+3b)(a-3b)$

(15) $-ab^2 + a = -a(b^2 - 1) = -a(b+1)(b-1)$

(16) $x^3y + 4x^2y + 4xy = xy(x^2 + 4x + 4) = xy(x+2)^2$

9

(解説)

(1) $x^4 - 10000 = (x^2)^2 - 100^2$
$\qquad\qquad = (x^2 + 100)(x^2 - 100)$
$\qquad\qquad = (x^2 + 100)(x+10)(x-10)$

(2) $y^4 - 256 = (y^2)^2 - 16^2$
$\qquad\qquad = (y^2 + 16)(y^2 - 16)$
$\qquad\qquad = (y^2 + 16)(y+4)(y-4)$

(3) $16a^4 - b^4 = (4a^2)^2 - (b^2)^2$
$\qquad\qquad = (4a^2 + b^2)(4a^2 - b^2)$
$\qquad\qquad = (4a^2 + b^2)(2a+b)(2a-b)$

(4) $81x^4 - 16y^4 = (9x^2)^2 - (4y^2)^2$
$\qquad\qquad = (9x^2 + 4y^2)(9x^2 - 4y^2)$
$\qquad\qquad = (9x^2 + 4y^2)(3x+2y)(3x-2y)$

(5) $625x^4 - 16y^4 = (25x^2)^2 - (4y^2)^2$
$\qquad\qquad = (25x^2 + 4y^2)(25x^2 - 4y^2)$
$\qquad\qquad = (25x^2 + 4y^2)(5x+2y)(5x-2y)$

(6) $256p^4 - 81q^4 = (16p^2)^2 - (9q^2)^2$
$\qquad\qquad = (16p^2 + 9q^2)(16p^2 - 9q^2)$
$\qquad\qquad = (16p^2 + 9q^2)(4p+3q)(4p-3q)$

10

(解説)

(1) $x^4 + 5x^2 - 6 = (x^2 - 1)(x^2 + 6)$
$\qquad\qquad = (x+1)(x-1)(x^2 + 6)$

(2) $a^4 - 13a^2 - 48 = (a^2 + 3)(a^2 - 16)$
$\qquad\qquad = (a^2 + 3)(a+4)(a-4)$

(3) $x^4 - 29x^2 + 100 = (x^2 - 4)(x^2 - 25)$
$\qquad\qquad = (x+2)(x-2)(x+5)(x-5)$

(4) $x^4 - 18x^2y^2 + 81y^4 = (x^2 - 9y^2)^2$
$\qquad\qquad = \{(x+3y)(x-3y)\}^2$
$\qquad\qquad = (x+3y)^2(x-3y)^2$

(5) $2x^4 - 17x^2 - 9 = (x^2 - 9)(2x^2 + 1)$
$\qquad\qquad = (x+3)(x-3)(2x^2 + 1)$

(6) $4a^4 - 25a^2b^2 + 36b^4 = (a^2 - 4b^2)(4a^2 - 9b^2)$
$\qquad\qquad = (a+2b)(a-2b)(2a+3b)(2a-3b)$

解説

(1) $(x+3)^2+7(x+3)+10=\{(x+3)+2\}\{(x+3)+5\}$
$\qquad =(x+5)(x+8)$

(2) $(a-4)^2-3(a-4)-18=\{(a-4)+3\}\{(a-4)-6\}$
$\qquad =(a-1)(a-10)$

(3) $(a-2b)^2-11(a-2b)+30=\{(a-2b)-5\}\{(a-2b)-6\}$
$\qquad =(a-2b-5)(a-2b-6)$

(4) $(x+y)^2+6(x+y)+9=\{(x+y)+3\}^2$
$\qquad =(x+y+3)^2$

(5) $6(x+y)^2-5(x+y)-4=\{2(x+y)+1\}\{3(x+y)-4\}$
$\qquad =(2x+2y+1)(3x+3y-4)$

(6) $8(x-y)^2-22(x-y)+15=\{2(x-y)-3\}\{4(x-y)-5\}$
$\qquad =(2x-2y-3)(4x-4y-5)$

(7) $(x-y)^2-4(x-y)z+4z^2=\{(x-y)-(2z)\}^2$
$\qquad =(x-y-2z)^2$

(8) $27(x+1)^2+15(x+1)y+2y^2=\{3(x+1)+y\}\{9(x+1)+2y\}$
$\qquad =(3x+y+3)(9x+2y+9)$

解説

(1) $(x^2+x)^2-8(x^2+x)+12=\{(x^2+x)-2\}\{(x^2+x)-6\}$
$\qquad =(x^2+x-2)(x^2+x-6)$
$\qquad =(x-1)(x+2)(x-2)(x+3)$

(2) $(x^2-7x)^2+4(x^2-7x)-12=\{(x^2-7x)-2\}\{(x^2-7x)+6\}$
$\qquad =(x^2-7x-2)(x^2-7x+6)$
$\qquad =(x^2-7x-2)(x-1)(x-6)$

(3) $(x^2+4x)^2+8(x^2+4x)+16=\{(x^2+4x)+4\}^2$
$\qquad =(x^2+4x+4)^2$
$\qquad =\{(x+2)^2\}^2$
$\qquad =(x+2)^4$

(4) $(a^2+2ab)^2-14b^2(a^2+2ab)-15b^4=\{(a^2+2ab)+b^2\}\{(a^2+2ab)-15b^2\}$
$\qquad =(a^2+2ab+b^2)(a^2+2ab-15b^2)$
$\qquad =(a+b)^2(a-3b)(a+5b)$

解説

(1) $(x-1)(x-3)(x-5)(x-7)+15=\{(x-1)(x-7)\}\{(x-3)(x-5)\}+15$
$\qquad =\{(x^2-8x)+7\}\{(x^2-8x)+15\}+15$
$\qquad =(x^2-8x)^2+22(x^2-8x)+120$
$\qquad =\{(x^2-8x)+12\}\{(x^2-8x)+10\}$
$\qquad =(x^2-8x+12)(x^2-8x+10)$
$\qquad =(x-2)(x-6)(x^2-8x+10)$

(2) $(x+1)(x-1)(x-3)(x-5)+12=\{(x+1)(x-5)\}\{(x-1)(x-3)\}+12$
$\qquad =\{(x^2-4x)-5\}\{(x^2-4x)+3\}+12$

$\qquad =(x^2-4x)^2-2(x^2-4x)-3$
$\qquad =\{(x^2-4x)+1\}\{(x^2-4x)-3\}$
$\qquad =(x^2-4x+1)(x^2-4x-3)$

(3) $(x-3)(x-1)(x+3)(x+5)+35=\{(x-3)(x+5)\}\{(x-1)(x+3)\}+35$
$\qquad =\{(x^2+2x)-15\}\{(x^2+2x)-3\}+35$
$\qquad =(x^2+2x)^2-18(x^2+2x)+80$
$\qquad =\{(x^2+2x)-8\}\{(x^2+2x)-10\}$
$\qquad =(x^2+2x-8)(x^2+2x-10)$
$\qquad =(x-2)(x+4)(x^2+2x-10)$

(4) $(x-4)(x-2)(x+1)(x+3)+24=(x-4)(x+3)\times(x-2)(x+1)+24$
$\qquad =(x^2-x-12)(x^2-x-2)+24$
$\qquad =(x^2-x)^2-14(x^2-x)+48$
$\qquad =\{(x^2-x)-6\}\{(x^2-x)-8\}$
$\qquad =(x^2-x-6)(x^2-x-8)$
$\qquad =(x+2)(x-3)(x^2-x-8)$

(5) $(x-1)(x-2)(x+3)(x+4)-84=(x-1)(x+3)\times(x-2)(x+4)-84$
$\qquad =(x^2+2x-3)(x^2+2x-8)-84$
$\qquad =(x^2+2x)^2-11(x^2+2x)+24-84$
$\qquad =(x^2+2x)^2-11(x^2+2x)-60$
$\qquad =\{(x^2+2x)-15\}\{(x^2+2x)+4\}$
$\qquad =(x-3)(x+5)(x^2+2x+4)$

(6) $(x+1)(x+2)(x+9)(x+10)-180=(x+1)(x+10)\times(x+2)(x+9)-180$
$\qquad =\{(x^2+11x)+10\}\{(x^2+11x)+18\}-180$
$\qquad =(x^2+11x)^2+28(x^2+11x)+180-180$
$\qquad =(x^2+11x)^2+28(x^2+11x)$
$\qquad =(x^2+11x)(x^2+11x+28)=x(x+11)(x+4)(x+7)$

(7) $(x-1)(x-3)(x-5)(x-7)-9=(x-1)(x-7)\times(x-3)(x-5)-9$
$\qquad =(x^2-8x+7)(x^2-8x+15)-9$
$\qquad =(x^2-8x)^2+22(x^2-8x)+96$
$\qquad =(x^2-8x+6)(x^2-8x+16)=(x-4)^2(x^2-8x+6)$

(8) $(x+1)(x+2)(x+3)(x+4)+1=\{(x+1)(x+4)\}\{(x+2)(x+3)\}+1$
$\qquad =\{(x^2+5x)+4\}\{(x^2+5x)+6\}+1$
$\qquad =(x^2+5x)^2+10(x^2+5x)+25=(x^2+5x+5)^2$

解説

(1) $xy-x+y-1=x(y-1)+(y-1)$
$\qquad =(x+1)(y-1)$

(2) $ab-2a-b+2=a(b-2)-(b-2)$
$\qquad =(a-1)(b-2)$

(3) $ab+bc-cd-da=b(a+c)-d(c+a)$
$\qquad =(a+c)(b-d)$

(4) $xy+yz+zu+ux=y(x+z)+u(z+x)$
$\qquad =(x+z)(y+u)$

(5) $2ax-bx+4ay-2by=x(2a-b)+2y(2a-b)$

$\qquad =(2a-b)(x+2y)$

(6) $6ap-15aq+4bp-10bq=3a(2p-5q)+2b(2p-5q)$
$\qquad =(3a+2b)(2p-5q)$

(7) $ax-bx+x-ay+by-y=x(a-b+1)-y(a-b+1)$
$\qquad =(a-b+1)(x-y)$

(8) $ax-bx-2ay+2by+3az-3bz=x(a-b)-2y(a-b)+3z(a-b)$
$\qquad =(a-b)(x-2y+3z)$

解説

(1) $x^2+6x+9-y^2=(x^2+6x+9)-y^2$
$\qquad =(x+3)^2-y^2$
$\qquad =\{(x+3)+y\}\{(x+3)-y\}$
$\qquad =(x+y+3)(x-y+3)$

(2) $x^2+14x+49-36y^2=(x^2+14x+49)-36y^2$
$\qquad =(x+7)^2-(6y)^2$
$\qquad =\{(x+7)+6y\}\{(x+7)-6y\}$
$\qquad =(x+6y+7)(x-6y+7)$

(3) $a^2+20a+100-81b^2=(a^2+20a+100)-81b^2$
$\qquad =(a+10)^2-(9b)^2$
$\qquad =\{(a+10)+9b\}\{(a+10)-9b\}$
$\qquad =(a+9b+10)(a-9b+10)$

(4) $9x^2-24x+16-25y^2=(9x^2-24x+16)-25y^2$
$\qquad =(3x-4)^2-(5y)^2$
$\qquad =\{(3x-4)+5y\}\{(3x-4)-5y\}$
$\qquad =(3x+5y-4)(3x-5y-4)$

(5) $a^2+2ab+b^2-x^2+6xy-9y^2=(a^2+2ab+b^2)-(x^2-6xy+9y^2)$
$\qquad =(a+b)^2-(x-3y)^2$
$\qquad =\{(a+b)+(x-3y)\}\{(a+b)-(x-3y)\}$
$\qquad =(a+b+x-3y)(a+b-x+3y)$

(6) $16x^2-40xy+25y^2-9z^2-42zw-49w^2$
$\quad =(16x^2-40xy+25y^2)-(9z^2+42zw+49w^2)$
$\quad =(4x-5y)^2-(3z+7w)^2$
$\quad =\{(4x-5y)+(3z+7w)\}\{(4x-5y)-(3z+7w)\}$
$\quad =(4x-5y+3z+7w)(4x-5y-3z-7w)$

1

(解説)

(1) （与式）$= mab \cdot m - mab \cdot a = mab(m-a)$

(2) （与式）$= 6x^2 \cdot x^2 + 6x^2 \cdot 2 = 6x^2(x^2+2)$

(3) （与式）$= 3ac \cdot 3b^2 - 3ac \cdot 2ac - 3ac \cdot bc = 3ac(3b^2 - 2ac - bc)$

(4) （与式）$= x \cdot x + 2x(y+z) = x\{x+2(y+z)\} = x(x+2y+2z)$

(5) （与式）$= x(a+b) + (-y)(a+b) = (a+b)(x-y)$

(6) （与式）$= 2a(a-3b) + b(a-3b) = (a-3b)(2a+b)$

2

(解説)

(1) $2x^2 + 5x = x \times 2x + x \times 5$
$\qquad = x(2x+5)$

(2) $9a^2 - 12ab = 3a \times 3a - 3a \times 4b$
$\qquad = 3a(3a-4b)$

(3) $3x^2y^2 - 2xy^2 + 4xy = xy \times 3xy - xy \times 2y + xy \times 4$
$\qquad = xy(3xy - 2y + 4)$

(4) $x^2 + 7x + 10 = x^2 + (2+5)x + 2 \times 5$
$\qquad = (x+2)(x+5)$

(5) $a^2 + a - 20 = a^2 + \{(-4)+5\}a + (-4) \times 5$
$\qquad = (a-4)(a+5)$

(6) $x^2 - 12x + 35 = x^2 + \{(-5)+(-7)\}x + (-5) \times (-7)$
$\qquad = (x-5)(x-7)$

(7) $y^2 - 3y - 28 = y^2 + \{4+(-7)\}y + 4 \times (-7)$
$\qquad = (y+4)(y-7)$

(8) $x^2 + 2x + 1 = x^2 + 2 \times 1 \times x + 1^2$
$\qquad = (x+1)^2$

(9) $b^2 - 12b + 36 = b^2 - 2 \times 6 \times b + 6^2$
$\qquad = (b-6)^2$

(10) $x^2 - 49 = x^2 - 7^2$
$\qquad = (x+7)(x-7)$

(11) $3a^2b + 18ab + 27b = 3b(a^2 + 6a + 9)$
$\qquad = 3b(a+3)^2$

(12) $64s^2 - 49t^2 = (8s)^2 - (7t)^2$
$\qquad = (8s+7t)(8s-7t)$

3

(解説)

(1) （与式）$= 5a(a-4) + 2(a-4) = (a-4)(5a+2)$

(2) （与式）$= (4x)^2 - 2 \cdot 4x \cdot 3y + (3y)^2 = (4x-3y)^2$

(3) （与式）$= 2^2 - (a-b)^2 = \{2+(a-b)\}\{2-(a-b)\} = (2+a-b)(2-a+b)$

(4) （与式）$= x^2 + (-6y-9y)x + (-6y) \cdot (-9y) = (x-6y)(x-9y)$

(5) （与式）$= (2a-7b)(3a+2b)$

$$\begin{array}{ccc} 2 & \diagdown & -7b \longrightarrow -21b \\ 3 & \diagup & 2b \longrightarrow 4b \\ \hline 6 & -14b^2 & -17b \end{array}$$

(6) （与式）$= (x^2)^2 + 5 \cdot x^2 - 9 \cdot 4 = (x^2+9)(x^2-4) = (x^2+9)(x+2)(x-2)$

4

(解説)

(1) $4a^2 - 49b^2 = (2a)^2 - (7b)^2 = (2a+7b)(2a-7b)$

(2) $9x^2 + 42x + 49 = (3x)^2 + 2 \times 3x \times 7 + 7^2 = (3x+7)^2$

(3) $x^2 - 9x + 18 = (x-3)(x-6)$

(4) $a(b-c) - b + c = a(b-c) - (b-c) = (a-1)(b-c)$

(5) $81 - 30t + t^2 = t^2 - 30t + 81 = (t-3)(t-27)$

(6) $35x^2 - 12xy + y^2 = y^2 - 12xy + 35x^2 = (y-5x)(y-7x)$
$\qquad = (5x-y)(7x-y)$

(7) $3a^2b - 6ab - 9b = 3b(a^2 - 2a - 3) = 3b(a+1)(a-3)$

(8) $4x^2 + 16x + 16 = 4(x^2 + 4x + 4) = 4(x+2)^2$

(9) $36a^2 - 4 = 4(9a^2 - 1) = 4(3a+1)(3a-1)$

(10) $\frac{1}{3}x^2 - 2x + 3 = \frac{1}{3}(x^2 - 6x + 9) = \frac{1}{3}(x-3)^2$

(11) $a(a-b+c) + c(b-a-c) = a(a-b+c) - c(a-b+c)$
$\qquad = (a-c)(a-b+c)$

(12) $5a^2x^3y + 30a^2x^2y^2 + 45a^2xy^3 = 5a^2xy(x^2 + 6xy + 9y^2)$
$\qquad = 5a^2xy(x+3y)^2$

5

(解説)

(1) $(x+1)^2 - (x+1) - 2 = \{(x+1)+1\}\{(x+1)-2\}$
$\qquad = (x+2)(x-1)$

(2) $a^2 + 2ab + b^2 - c^2 = (a^2 + 2ab + b^2) - c^2$
$\qquad = (a+b)^2 - c^2$
$\qquad = \{(a+b)+c\}\{(a+b)-c\}$
$\qquad = (a+b+c)(a+b-c)$

(3) $(x+y-1)(x+y+3) - 5 = (x+y)^2 + 2(x+y) - 8$
$\qquad = \{(x+y)-2\}\{(x+y)+4\}$
$\qquad = (x+y-2)(x+y+4)$

6

(解説)

(1) $\frac{1}{4}x^2 + x + 1 = \left(\frac{1}{2}x\right)^2 + 2 \cdot \frac{1}{2}x \cdot 1 + 1^2 = \left(\frac{1}{2}x+1\right)^2$

(2) $27xy^3 - 12x^3y = 3xy(9y^2 - 4x^2)$
$\qquad = 3xy(3y+2x)(3y-2x)$

(3) $(3x-2)^2 - 2(3x+2) = 9x^2 - 12x + 4 - 6x - 4$
$\qquad = 9x^2 - 18x = 9x(x-2)$

(4) $3a^4b - 4a^2b^3 - a^3b^2 = a^2b(3a^2 - 4b^2 - ab)$
$\qquad = a^2b(3a^2 - ab - 4b^2)$
$\qquad = a^2b(a+b)(3a-4b)$

$$\begin{array}{ccc} 1 & \diagdown & b \longrightarrow 3b \\ 3 & \diagup & -4b \longrightarrow -4b \\ \hline 3 & -4b^2 & -b \end{array}$$

(5) $a^2(a-b) + 9b^2(b-a) = a^2(a-b) - 9b^2(a-b)$
$\qquad = (a-b)(a^2 - 9b^2)$
$\qquad = (a-b)(a+3b)(a-3b)$

(6) $(x+y)^2 - x - y - 2 = (x+y)^2 - (x+y) - 2$
$\qquad = \{(x+y)+1\}\{(x+y)-2\}$
$\qquad = (x+y+1)(x+y-2)$

(7) $9x^2 + 6xy + y^2 - 25 = (9x^2 + 6xy + y^2) - 25$
$\qquad = (3x+y)^2 - 5^2$
$\qquad = (3x+y+5)(3x+y-5)$

(8) $x^2 - y^2 - 4y - 4 = x^2 - (y^2 + 4y + 4)$
$\qquad = x^2 - (y+2)^2$
$\qquad = \{x+(y+2)\}\{x-(y+2)\}$
$\qquad = (x+y+2)(x-y-2)$

7

(解説)

(1) $x^4 - 810000 = (x^2)^2 - 900^2 = (x^2+900)(x^2-900)$
$\qquad = (x^2+900)(x+30)(x-30)$

(2) $16a^4 - 625b^4 = (4a^2)^2 - (25b^2)^2 = (4a^2 + 25b^2)(4a^2 - 25b^2)$
$\qquad = (4a^2 + 25b^2)(2a+5b)(2a-5b)$

(3) $x^4 - (4x^2 - 12x + 9) = (x^2)^2 - (2x-3)^2$
$\qquad = \{x^2 + (2x-3)\}\{x^2 - (2x-3)\}$
$\qquad = (x^2 + 2x - 3)(x^2 - 2x + 3)$
$\qquad = (x+3)(x-1)(x^2 - 2x + 3)$

(4) $(3a-b)^2 - (5b-a)^2 = \{(3a-b)+(5b-a)\}\{(3a-b)-(5b-a)\}$
$\qquad = (2a+4b)(4a-6b)$
$\qquad = 4(a+2b)(2a-3b)$

(5) $(a-5)^2 - 2(a-5) - 24 = \{(a-5)+4\}\{(a-5)-6\}$
$\qquad = (a-1)(a-11)$

8

(解説)

(1) $a+b=t$ とおくと
$$2(a+b)^2-(a+b)-6=2t^2-t-6=(t-2)(2t+3)$$
$$=\{(a+b)-2\}\{2(a+b)+3\}$$
$$=(a+b-2)(2a+2b+3)$$

(2) $x+2y=t$ とおくと
$$(x+2y)^2-3(x+2y)z-4z^2=t^2-3tz-4z^2=(t+z)(t-4z)$$
$$=\{(x+2y)+z\}\{(x+2y)-4z\}$$
$$=(x+2y+z)(x+2y-4z)$$

(3) $16x^2-9y^2+12y-4=16x^2-(9y^2-12y+4)$
$$=16x^2-(3y-2)^2$$
ここで，$3y-2=t$ とおくと
$$16x^2-9y^2+12y-4=16x^2-t^2=(4x+t)(4x-t)$$
$$=\{4x+(3y-2)\}\{4x-(3y-2)\}$$
$$=(4x+3y-2)(4x-3y+2)$$

(4) $x^4-256=(x^2)^2-16^2=(x^2+16)(x^2-16)$
$$=(x^2+16)(x+4)(x-4)$$

(5) $a^2=A$，$b^2=B$ とおくと
$$9a^4-37a^2b^2+4b^4=9A^2-37AB+4B^2=(A-4B)(9A-B)$$
$$=(a^2-4b^2)(9a^2-b^2)$$
$$=(a+2b)(a-2b)(3a+b)(3a-b)$$

9

(解説)

(1) $(x+y+1)^2-(x-y)^2=\{(x+y+1)+(x-y)\}\{(x+y+1)-(x-y)\}$
$$=(2x+1)(2y+1)$$

(2) $(x+1)^2+2(x+1)-8=\{(x+1)+4\}\{(x+1)-2\}$
$$=(x+5)(x-1)$$

(3) $(x+1)^2-2(x+1)-3=\{(x+1)+1\}\{(x+1)-3\}$
$$=(x+2)(x-2)$$

(4) $(x^2-2x)^2-4(x^2-2x)+3=\{(x^2-2x)-1\}\{(x^2-2x)-3\}$
$$=(x^2-2x-1)(x^2-2x-3)$$
$$=(x^2-2x-1)(x+1)(x-3)$$

(5) $(x^2-6x)^2+(x^2-6x)-56=\{(x^2-6x)+8\}\{(x^2-6x)-7\}$
$$=(x^2-6x+8)(x^2-6x-7)$$
$$=(x-2)(x-4)(x+1)(x-7)$$

(6) $(x^2+4x)^2-8(x^2+4x)-48=\{(x^2+4x)+4\}\{(x^2+4x)-12\}$
$$=(x^2+4x+4)(x^2+4x-12)$$
$$=(x+2)^2(x+6)(x-2)$$

10

(解説)

(1) （与式）$=(x^2)^2-10x^2+9=(x^2-1)(x^2-9)$
$$=(x+1)(x-1)(x+3)(x-3)$$

(2) （与式）$=\{(x^2-2x)-3\}\{(x^2-2x)-8\}$
$$=(x^2-2x-3)(x^2-2x-8)$$
$$=(x+1)(x-3)(x+2)(x-4)$$
$$=(x+1)(x+2)(x-3)(x-4)$$

(3) （与式）$=(x^2+2)^2-(2x)^2$
$$=\{(x^2+2)+2x\}\{(x^2+2)-2x\}$$
$$=(x^2+2x+2)(x^2-2x+2)$$

(4) （与式）$=(x+3)^3-4(x+3)=(x+3)\{(x+3)^2-4\}$
$$=(x+3)\{(x+3)+2\}\{(x+3)-2\}$$
$$=(x+3)(x+5)(x+1)=(x+1)(x+3)(x+5)$$

11

(解説)

(1) $a^2+2ab+b^2=(a+b)^2$
$$=(58+42)^2$$
$$=100^2$$
$$=10000$$

(2) $a^2-b^2=(a+b)(a-b)$
$$=(58+42)\times(58-42)$$
$$=100\times16$$
$$=1600$$

(3) $a^2+ab-8a-8b=a(a+b)-8(a+b)$
$$=(a-8)(a+b)$$
$$=(58-8)\times(58+42)$$
$$=50\times100$$
$$=5000$$

(4) $ab-2a-8b+16=a(b-2)-8(b-2)$
$$=(a-8)(b-2)$$
$$=(58-8)\times(42-2)$$
$$=50\times40$$
$$=2000$$

1

(解説)

(1) $xy-x-y+1=(y-1)x-y+1$
$$=(y-1)x-(y-1)=(x-1)(y-1)$$

(2) $a^2-2a^2b+2b-a=(-2a^2+2)b+a^2-a$
$$=-2(a+1)(a-1)b+a(a-1)$$
$$=(a-1)\{-2(a+1)b+a\}$$
$$=(a-1)(-2ab+a-2b)$$

(3) $ab^2-b^2c-c^2a+bc^2=(b^2-c^2)a-b^2c+bc^2$
$$=(b+c)(b-c)a-bc(b-c)$$
$$=(b-c)\{(b+c)a-bc\}$$
$$=(b-c)(ab-bc+ca)$$

(4) $x^2-2xy+4x-2y+3=-2(x+1)y+(x+1)(x+3)$
$$=(x+1)(-2y+x+3)$$
$$=(x+1)(x-2y+3)$$

2

(解説)

(1) $x^3-y^3+x^2y-xy^2=(x^3+x^2y)-(xy^2+y^3)$
$$=x^2(x+y)-y^2(x+y)$$
$$=(x+y)(x^2-y^2)$$
$$=(x+y)(x+y)(x-y)$$
$$=(x+y)^2(x-y)　答$$

(2) $2xy-2yz+2zx-x^2-y^2=-(x^2-2xy+y^2)+(2zx-2yz)$
$$=-(x-y)^2+2z(x-y)$$
$$=(x-y)\{-(x-y)+2z\}$$
$$=(x-y)(-x+y+2z)　答$$

3

(解説)

(1) $(x^2+2x)^2-2x^2-4x-3$
$$=(x^2+2x)^2-2(x^2+2x)-3$$
$$=\{(x^2+2x)+1\}\{(x^2+2x)-3\}$$
$$=(x^2+2x+1)(x^2+2x-3)$$
$$=(x+1)^2(x-1)(x+3)$$

(2) $x^2-y^2-z^2+2x+2yz+1$
$$=(x^2+2x+1)-(y^2-2yz+z^2)$$
$$=(x+1)^2-(y-z)^2$$
$$=\{(x+1)+(y-z)\}\{(x+1)-(y-z)\}$$
$$=(x+y-z+1)(x-y+z+1)$$

[4]

(解説)

(1) $ac+bc+2a+2b-3c-6=(ac+bc-3c)+(2a+2b-6)$
$=c(a+b-3)+2(a+b-3)$
$=(a+b-3)(c+2)$

(2) $x^2-y^2-x+y=(x^2-y^2)-(x-y)=(x+y)(x-y)-(x-y)$
$=(x-y)(x+y-1)$

(3) $xy^2-y-x^2y+x=(xy^2-x^2y)+(x-y)$
$=xy(y-x)+(x-y)$
$=-xy(x-y)+(x-y)$
$=(x-y)(-xy+1)$

(4) $1-x^2-2xy-y^2=1-(x^2+2xy+y^2)$
$=1-(x+y)^2$
$=\{1+(x+y)\}\{1-(x+y)\}$
$=(x+y+1)(-x-y+1)$

(5) $x^3-x^2-x+1=(x^3-x^2)-(x-1)$
$=x^2(x-1)-(x-1)=(x-1)(x^2-1)$
$=(x-1)(x+1)(x-1)=(x+1)(x-1)^2$

(6) $a^2-2ab+b^2-6a+6b=(a^2-2ab+b^2)-(6a-6b)$
$=(a-b)^2-6(a-b)$
$=(a-b)\{(a-b)-6\}$
$=(a-b)(a-b-6)$

[5]

(解説)

(1) $x^2-xy+zx-wx+wy-wz=(x-w)z+x^2-xy-wx+wy$
$=(x-w)z+(x-w)x-(x-w)y$
$=(x-w)(x-y+z)$

(2) $x^2+2xy+y^2-3x-3y+2=(x+y)^2-3(x+y)+2$
$=(x+y-1)(x+y-2)$

(3) $x^2-y^2-x+5y-6=x^2-x-(y^2-5y+6)$
$=x^2-x-(y-2)(y-3)$
$=\{x-(y-2)\}\{x+(y-3)\}$
$=(x-y+2)(x+y-3)$

(4) $x^3+2x^2-4x-8=x^2(x+2)-4(x+2)$
$=(x+2)(x^2-4)$
$=(x+2)(x+2)(x-2)$
$=(x+2)^2(x-2)$

[6]

(解説)

(1) $9x(x-2)-4y(y-3)=9x^2-18x-4y^2+12y$
$=(9x^2-4y^2)-(18x-12y)$
$=(3x+2y)(3x-2y)-6(3x-2y)$
$=(3x-2y)(3x+2y-6)$

(2) $(x+2y)(x-2y)-4y-1=x^2-4y^2-4y-1$

$=x^2-(4y^2+4y+1)$
$=x^2-(2y+1)^2$
$=\{x+(2y+1)\}\{x-(2y+1)\}$
$=(x+2y+1)(x-2y-1)$

(3) $(x+y-3)(x-y-3)+12x=\{(x-3)+y\}\{(x-3)-y\}+12x$
$=(x-3)^2-y^2+12x$
$=x^2-6x+9+12x-y^2$
$=x^2+6x+9-y^2$
$=(x+3)^2-y^2$
$=\{(x+3)+y\}\{(x+3)-y\}$
$=(x+y+3)(x-y+3)$

(4) $a^2-ab-4a+2b+4=(-ab+2b)+(a^2-4a+4)$
$=-b(a-2)+(a-2)^2$
$=(a-2)(-b+a-2)=(a-2)(a-b-2)$

(5) $x^2z-z^3-xyz+yz^2=z(x^2-z^2-xy+yz)$
$=z\{(x+z)(x-z)-y(x-z)\}$
$=z(x-z)\{(x+z)-y\}$
$=z(x-z)(x-y+z)$

(6) $2(a-b)^2(a+b)-(a-b)(a^2+2b^2)=(a-b)\{2(a-b)(a+b)-(a^2+2b^2)\}$
$=(a-b)\{2(a^2-b^2)-a^2-2b^2\}$
$=(a-b)(a^2-4b^2)$
$=(a-b)(a+2b)(a-2b)$

[7]

(解説)

(1) $(3x+y)^2-(x+2y)(3x+y)-(2x-y)(2x-3y)$
$=(3x+y)\{(3x+y)-(x+2y)\}-(2x-y)(2x-3y)$
$=(3x+y)(2x-y)-(2x-y)(2x-3y)$
$=(2x-y)\{(3x+y)-(2x-3y)\}$
$=(2x-y)(x+4y)$

(2) $(x+y-3)(x-y-3)+12x=\{(x-3)+y\}\{(x-3)-y\}+12x$
$=(x-3)^2-y^2+12x$
$=x^2-6x+9-y^2+12x$
$=x^2+6x+9-y^2$
$=(x+3)^2-y^2$
$=\{(x+3)+y\}\{(x+3)-y\}$
$=(x+y+3)(x-y+3)$

(3) $a^3b+16-4ab-4a^2=a^3b-4ab-4a^2+16$
$=ab(a^2-4)-4(a^2-4)$
$=(a^2-4)(ab-4)$
$=(a+2)(a-2)(ab-4)$

(4) $(ab+2)^2+(2a-b)^2=a^2b^2+4ab+4+4a^2-4ab+b^2$
$=a^2b^2+4a^2+b^2+4$
$=a^2(b^2+4)+(b^2+4)$
$=(a^2+1)(b^2+4)$

---

(5) $2ab+2bc-2ca-a^2-b^2=2bc-2ca-a^2+2ab-b^2$
$=2c(b-a)-(a^2-2ab+b^2)$
$=-2c(a-b)-(a-b)^2$
$=(a-b)\{-2c-(a-b)\}$
$=(a-b)(-a+b-2c)$

(6) $x^2(y-z)+y^2(z-x)=x^2y-x^2z+y^2z-xy^2$
$=(y^2-x^2)z-xy(y-x)$
$=(y-x)(y+x)z-xy(y-x)$
$=(y-x)\{(y+x)z-xy\}$
$=(y-x)(-xy+yz+zx)$
$=(x-y)(xy-yz-zx)$

[8] [広島国際学院大]

(解説)

$(ax-3y)^2-(ay-3x)^2=(ax-3y+ay-3x)(ax-3y-ay+3x)$
$=\{a(x+y)-3(x+y)\}\{a(x-y)+3(x-y)\}$
$=(a-3)(x+y)(a+3)(x-y)$
$=(a+3)(a-3)(x+y)(x-y)$　圏

[9] [昭和学院秀英]

(解説)

$\begin{cases} a+b=14 & \cdots\cdots ① \\ a^2-b^2=56 & \cdots\cdots ② \end{cases}$

②より　　$(a+b)(a-b)=56$

①を代入すると　$14(a-b)=56$
$a-b=4$　$\cdots\cdots ③$

①+③より　　$2a=18$
$a=9$

$a=9$ を①に代入すると
$9+b=14$
$b=5$

これらは問題に適している。
よって　　$a=9$,　$b=5$

[10]

(解説)

$\dfrac{1}{a}+\dfrac{1}{b}=-3$ の両辺に ab をかけると　　$b+a=-3ab$

よって　　$\dfrac{a^2b-ab^2}{a^2-b^2}=\dfrac{ab(a-b)}{(a+b)(a-b)}$
$=\dfrac{ab}{a+b}$
$=\dfrac{ab}{-3ab}$
$=-\dfrac{1}{3}$

11

(解説)

(1) $ax-bx-cx-ay+by+cy=x(a-b-c)-y(a-b-c)$
$$=(a-b-c)(x-y)$$
$$=5\times(-2)$$
$$=-10$$

(2) $x^2-2xy+y^2-4x+4y-2=(x^2-2xy+y^2)-4(x-y)-2$
$$=(x-y)^2-4(x-y)-2$$
$$=(-3)^2-4\times(-3)-2$$
$$=9+12-2$$
$$=19$$

(3) $a^2+b^2-2ab-3a+3b+3=(a^2-2ab+b^2)-3(a-b)+3$
$$=(a-b)^2-3(a-b)+3$$
$$=1^2-3\times1+3$$
$$=1-3+3$$
$$=1$$

(4) $x^2y-xy^2+x-y=(x^2y-xy^2)+(x-y)$
$$=xy(x-y)+(x-y)$$
$$=(xy+1)(x-y)$$
$$=(4+1)(x-y)$$
$$=5(x-y)$$

であるから　$5(x-y)=10$
よって　　　$x-y=2$
したがって

$x^3y-2x^2y^2+xy^3=xy(x^2-2xy+y^2)$
$$=xy(x-y)^2$$
$$=4\times2^2$$
$$=16$$

(5) $2x^2+3xy-2y^2=(x+2y)(2x-y)$
$$=\frac{9}{2}\times(-1)$$
$$=-\frac{9}{2}$$

(参考)　この問題では，連立方程式 $\begin{cases}x+2y=\dfrac{9}{2}\\2x-y=-1\end{cases}$ を解かずに解答しているが，連立方

程式を解いて，上の解答が正しいことを確認することができる。

連立方程式の解は，$x=\dfrac{1}{2}$，$y=2$ となる。

これらを $2x^2+3xy-2y^2$ に代入すると
$$2\times\left(\frac{1}{2}\right)^2+3\times\frac{1}{2}\times2-2\times2^2=-\frac{9}{2}$$

となり，上の解答が正しいことが確認できる。

12

(解説)

縦に n 枚，横に $(n+2)$ 枚のタイルが並べられているとする。n は偶数である。
このとき，しきつめられているタイルの枚数は
$$n(n+2)\text{ 枚}$$
である。
よって，これより 1 枚多い枚数は　$n(n+2)+1$ 枚
ここで　　　　　$n(n+2)+1=n^2+2n+1$
$$=(n+1)^2$$
n は偶数であるから，$n+1$ は奇数となる。
したがって，$(n+1)^2$ は奇数の 2 乗であるから，この枚数のタイルを並べて正方形の床にしきつめることができる。

13 [山口県]

(解説)

(1) 6 の段には，2 の段に並んでいる 9 つの数をそれぞれ 3 倍した数が並んでいる。

よって，□ にあてはまる数は　3

(2) $a=n^2$ と表すとき，
$$b=n(n+1),\ c=n(n+1),\ d=(n+1)^2$$
と表すことができる。
よって　　$a+b+c+d=n^2+n(n+1)+n(n+1)+(n+1)^2$
$$=n^2+n^2+n+n^2+n+n^2+2n+1$$
$$=4n^2+4n+1$$
$$=(2n+1)^2$$
n が自然数のとき，$2n+1$ は奇数となるから，$a+b+c+d$ の値は奇数の 2 乗となる。
したがって，T さんの予想は正しい。

1

(解説)

(1) $ax^2+(ab+1)x+b=(x+b)(ax+1)$

(2) $3x^2+2(3a+b)x+4ab=(x+2a)(3x+2b)$

(1)
```
1   b  ⟶  ab
a   1  ⟶   1
─────────────
a   b    ab+1
```

(2)
```
1   2a  ⟶  6a
3   2b  ⟶  2b
──────────────
3   4ab   6a+2b
```

(3) $abx^2+(a^2-b^2)x-ab=(ax-b)(bx+a)$

(4) $2abx^2-(4a+3b)x+6=(2ax-3)(bx-2)$

(3)
```
a   -b  ⟶  -b^2
b    a  ⟶   a^2
─────────────────
ab  -ab   a^2-b^2
```

(4)
```
2a  -3  ⟶  -3b
b   -2  ⟶  -4a
────────────────
2ab   6   -4a-3b
```

2

(解説)

(1) (与式)$=(x+y)\{x+(y-1)\}$
$$=(x+y)(x+y-1)$$
```
1   y    ⟶    y
1   y-1  ⟶   y-1
──────────────────
1   y(y-1)   2y-1
```

(2) (与式)$=\{x-(y+2)\}\{x-(2y-1)\}$
$$=(x-y-2)(x-2y+1)$$
```
1   -(y+2)    ⟶   -(y+2)
1   -(2y-1)   ⟶  -(2y-1)
──────────────────────────
1   (y+2)(2y-1)   -(3y+1)
```

(3) (与式)$=x^2+(-y+2)x-(2y^2+7y+3)$
$$=x^2+(-y+2)x-(y+3)(2y+1)$$
$$=\{x+(y+3)\}\{x-(2y+1)\}$$
$$=(x+y+3)(x-2y-1)$$
```
1   y+3        ⟶    y+3
1   -(2y+1)    ⟶  -2y-1
──────────────────────────
1   -(y+3)(2y+1)   -y+2
```

(4) (与式)$=(ax+b)(bx+a)$
```
a   b   ⟶  b^2
b   a   ⟶  a^2
────────────────
ab  ab   a^2+b^2
```

(5) (与式)$=2x^2+(-3y+5)x-(2y^2-5y+3)$
$$=2x^2+(-3y+5)x-(y-1)(2y-3)$$
$$=\{x-(2y-3)\}\{2x+(y-1)\}$$
$$=(x-2y+3)(2x+y-1)$$
```
1   -(2y-3)    ⟶  -4y+6
2    y-1       ⟶    y-1
──────────────────────────
2   -(y-1)(2y-3)   -3y+5
```

(6) (与式)$=6x^2+(5y+1)x-(6y^2+5y+1)$
$$=6x^2+(5y+1)x-(2y+1)(3y+1)$$
$$=\{2x+(3y+1)\}\{3x-(2y+1)\}$$
$$=(2x+3y+1)(3x-2y-1)$$
```
2    3y+1      ⟶   9y+3
3   -(2y+1)    ⟶  -4y-2
──────────────────────────
6   -(2y+1)(3y+1)   5y+1
```

$\boxed{3}$ [鹿児島経済大]

（解説）

(1) $a(b+c)^2+b(c+a)^2+c(a+b)^2-4abc$

$=(b+c)^2a+b(c^2+2ca+a^2)+c(a^2+2ab+b^2)-4abc$

$=(b+c)^2a+\{(b+c)^2+2bc+2bc-4bc\}a+bc^2+b^2c$

$=(b+c)^2a+(b+c)^2a+bc(b+c)$

$=(b+c)\{a^2+(b+c)a+bc\}$

$=(b+c)(a+b)(a+c)$

$=(a+b)(b+c)(c+a)$

(2) $x(y^2-z^2)+y(z^2-x^2)+z(x^2-y^2)=-(y-z)x^2+(y^2-z^2)x+yz^2-y^2z$

$=-(y-z)x^2+(y+z)(y-z)x-yz(y-z)$

$=-(y-z)\{x^2-(y+z)x+yz\}$

$=-(y-z)(x-y)(x-z)$

$=(x-y)(y-z)(z-x)$

$\boxed{4}$

（解説）

(1) 与式 $=(x^4+4x^2+4)-x^2=(x^2+2)^2-x^2=\{(x^2+2)+x\}\{(x^2+2)-x\}$

$=(x^2+x+2)(x^2-x+2)$

(2) 与式 $=(x^4-2x^2+1)-4x^2=(x^2-1)^2-(2x)^2=\{(x^2-1)+2x\}\{(x^2-1)-2x\}$

$=(x^2+2x-1)(x^2-2x-1)$

(3) 与式 $=(x^4-2x^2y^2+y^4)-16x^2y^2=(x^2-y^2)^2-(4xy)^2$

$=\{(x^2-y^2)+4xy\}\{(x^2-y^2)-4xy\}=(x^2+4xy-y^2)(x^2-4xy-y^2)$

(4) 与式 $=(x^4+4x^2y^2+4y^4)-4x^2y^2=(x^2+2y^2)^2-(2xy)^2$

$=\{(x^2+2y^2)+2xy\}\{(x^2+2y^2)-2xy\}=(x^2+2xy+2y^2)(x^2-2xy+2y^2)$

$\boxed{5}$ [立命館大]

（解説）

$(a+b+c)(a^2+b^2+c^2-ab-bc-ca)=\{a+(b+c)\}\{a^2-(b+c)a+b^2-bc+c^2\}$

$=a^3+\{(b+c)-(b+c)\}a^2+\{-(b+c)^2+(b^2-bc+c^2)\}a+(b+c)(b^2-bc+c^2)$

$=a^3-3bca+b^3+c^3\overset{\mathcal{P}}{=}a^3+b^3+c^3-3abc$

$8x^3+27y^3+18xy-1$

$=(2x)^3+(3y)^3+(-1)^3-3\cdot2x\cdot3y\cdot(-1)$

よって，（ア）から

$8x^3+27y^3+18xy-1$

$=(2x+3y-1)\{(2x)^2+(3y)^2+(-1)^2-2x\cdot3y-3y\cdot(-1)-(-1)\cdot2x\}$

$=(2x+3y-1)(4x^2+9y^2+1-6xy+3y+2x)$

$\overset{\mathcal{A}}{=}(2x+3y-1)(4x^2-6xy+9y^2+2x+3y+1)$

$\boxed{6}$ [愛知工業大]

（解説）

(与式)$=a^2b+ab^2-2bc(b-c)+2c^2a-ca^2-3abc$

$=(b-c)a^2+(b^2-3bc+2c^2)a-2bc(b-c)$

$=(b-c)a^2+(b-c)(b-2c)a-2bc(b-c)$

$=(b-c)\{a^2+(b-2c)a-2bc\}=(b-c)(a+b)(a-2c)$

$=(a+b)(b-c)(a-2c)$

$\boxed{7}$ [東北福祉大]

（解説）

(1) $(ac+bd)^2+(ad-bc)^2=(a^2c^2+2abcd+b^2d^2)+(a^2d^2-2abcd+b^2c^2)$

$=a^2(c^2+d^2)+b^2(c^2+d^2)=(a^2+b^2)(c^2+d^2)$

(2) (1) の恒等式に $a^2+b^2=1$，$c^2+d^2=1$，$ac+bd=1$ を代入すると

$1^2+(ad-bc)^2=1\cdot1$　　すなわち　　$(ad-bc)^2=0$

よって　　$ad-bc=0$　　ゆえに　　$ad=bc$

この両辺を2乗すると　　$a^2d^2=b^2c^2$

これに $d^2=1-c^2$，$b^2=1-a^2$ を代入して　　$a^2(1-c^2)=(1-a^2)c^2$

すなわち　　$a^2=c^2$

よって　　$a^2+d^2=c^2+d^2=1$，　　$b^2+c^2=b^2+a^2=1$

（参考）$\vec{p}=(a,\ b)$，$\vec{q}=(c,\ d)$ とすると，条件式から　　$|\vec{p}|=1$，$|\vec{q}|=1$，$\vec{p}\cdot\vec{q}=1$

このとき，$\vec{p}=\vec{q}$ が成り立つから　　$a=c$ かつ $b=d$

よって　　$ad-bc=cd-dc=0$，　　$a^2+d^2=c^2+d^2=1$，　　$b^2+c^2=d^2+c^2=1$

$\boxed{8}$ [流通科学大]

（解説）

$\dfrac{x^2+2x-8}{x^2-x-20}\div\dfrac{x^2+x-6}{x^2-25}=\dfrac{(x-2)(x+4)}{(x+4)(x-5)}\times\dfrac{(x+5)(x-5)}{(x-2)(x+3)}=\dfrac{x+5}{x+3}$

$\boxed{9}$ [駒澤大]

（解説）

(与式)$=\dfrac{x+11}{(x+3)(2x+1)}-\dfrac{x-10}{(x-2)(2x+1)}=\dfrac{(x+11)(x-2)-(x-10)(x+3)}{(x+3)(x-2)(2x+1)}$

$=\dfrac{(x^2+9x-22)-(x^2-7x-30)}{(x+3)(x-2)(2x+1)}=\dfrac{16x+8}{(x+3)(x-2)(2x+1)}$

$=\dfrac{8(2x+1)}{(x+3)(x-2)(2x+1)}=\dfrac{8}{(x+3)(x-2)}$

$\boxed{10}$ [慶應義塾]

（解説）

$\left(1-\dfrac{1}{2^2}\right)\left(1-\dfrac{1}{3^2}\right)\left(1-\dfrac{1}{4^2}\right)\times\ \cdots\cdots\ \times\left(1-\dfrac{1}{999^2}\right)$

$=\left(\dfrac{2^2-1^2}{2^2}\right)\left(\dfrac{3^2-1^2}{3^2}\right)\left(\dfrac{4^2-1^2}{4^2}\right)\times\ \cdots\cdots\ \times\left(\dfrac{999^2-1^2}{999^2}\right)$

$=\dfrac{(2-1)(2+1)}{2^2}\times\dfrac{(3-1)(3+1)}{3^2}\times\dfrac{(4-1)(4+1)}{4^2}\times\ \cdots\cdots\ \times\dfrac{(999-1)(999+1)}{999^2}$

$=\dfrac{1\times3}{2\times2}\times\dfrac{2\times4}{3\times3}\times\dfrac{3\times5}{4\times4}\times\ \cdots\cdots\ \times\dfrac{998\times1000}{999\times999}$

$=\dfrac{1\times1000}{2\times999}$

$=\dfrac{500}{999}$

$\boxed{11}$ [市川]

（解説）

$x^2=y^2+12$

$x^2-y^2=12$

$(x+y)(x-y)=12$

x，y は自然数であるから，$x+y$ は自然数，$x-y$ は整数である。

また，$x+y>x-y$ であり，和は $(x+y)+(x-y)=2x$ （偶数）

よって　　$\begin{cases}x+y=6\\x-y=2\end{cases}$

これを解くと　$(x,\ y)=(4,\ 2)$

$\boxed{12}$ [帝塚山]

（解説）

(1) $\dfrac{n(n+1)(n+2)}{3}-\dfrac{(n-1)n(n+1)}{3}=\dfrac{n(n+1)}{3}\{(n+2)-(n-1)\}$

$=\dfrac{n(n+1)}{3}\times3$

$=n(n+1)$

(2) (1) より　　$n(n+1)=\dfrac{1}{3}\{(n-1)n(n+1)-n(n+1)(n+2)\}$

よって　　$1\times2+2\times3+3\times4+\ \cdots\cdots\ +100\times101$

$=-\dfrac{1}{3}\times\{(0\times1\times2-1\times2\times3)+(1\times2\times3-2\times3\times4)$

$+(2\times3\times4-3\times4\times5)+\ \cdots\cdots\ +(99\times100\times101-100\times101\times102)\}$

$=-\dfrac{1}{3}\times(0\times1\times2-100\times101\times102)$

$=\dfrac{1}{3}\times100\times101\times102$

$=100\times101\times34$

$=343400$