

第8章 展開 例題

1

解説

- (1) $(x+2)(y-3) = xy - 3x + 2y - 6$
 (2) $(3a+4)(2a-1) = 6a^2 - 3a + 8a - 4 = 6a^2 + 5a - 4$
 (3) $(x-2y+3)(3x-y) = x(3x-y) - 2y(3x-y) + 3(3x-y) = 3x^2 - xy - 6xy + 2y^2 + 9x - 3y = 3x^2 - 7xy + 2y^2 + 9x - 3y$

2

解説

- (1) $(x+2)(x+7) = x^2 + (2+7)x + 2 \times 7 = x^2 + 9x + 14$
 (2) $(x+6)(x-4) = x^2 + (6-4)x + 6 \times (-4) = x^2 + 2x - 24$
 (3) $(y-2)(y-4) = y^2 + (-2-4)y + (-2) \times (-4) = y^2 - 6y + 8$
 (4) $(a-9)(a+3) = a^2 + (-9+3)a + (-9) \times 3 = a^2 - 6a - 27$
 (5) $(2x+7)(2x-9) = (2x)^2 + (7-9) \times 2x + 7 \times (-9) = 4x^2 - 4x - 63$

3

解説

- (1) $(x+6)^2 = x^2 + 2 \times 6 \times x + 6^2 = x^2 + 12x + 36$
 (2) $(3x-5)^2 = (3x)^2 - 2 \times 5 \times 3x + 5^2 = 9x^2 - 30x + 25$
 (3) $(2x+3y)^2 = (2x)^2 + 2 \times 2x \times 3y + (3y)^2 = 4x^2 + 12xy + 9y^2$

4

解説

- (1) $(x+6)(x-6) = x^2 - 6^2 = x^2 - 36$
 (2) $(2a-3b)(2a+3b) = (2a)^2 - (3b)^2 = 4a^2 - 9b^2$

5

解説

- (1) $(x-5)(x+7) = x^2 + ((-5)+7)x + (-5) \times 7 = x^2 + 2x - 35$
 (2) $(x-8)^2 = x^2 - 2 \times 8 \times x + 8^2 = x^2 - 16x + 64$
 (3) $(x+5y)(x-5y) = x^2 - (5y)^2 = x^2 - 25y^2$
 (4) $(2a+3b)^2 = (2a)^2 + 2 \times 3b \times 2a + (3b)^2 = 4a^2 + 12ab + 9b^2$
 (5) $(2x+y)(2x-y) = (2x)^2 - y^2 = 4x^2 - y^2$
 (6) $(3x-1)(3x-4) = (3x)^2 + ((-1)+(-4)) \times 3x + (-1) \times (-4) = 9x^2 - 15x + 4$

6

解説

- (1) $(x-1)^2 - (x+2)(x-8) = (x^2 - 2x + 1) - (x^2 - 6x - 16) = x^2 - 2x + 1 - x^2 + 6x + 16 = 4x + 17$
 (2) $3(x+2y)(x-2y) + (x-3y)^2 - (2x+3y)(2x-y) = 3(x^2 - 4y^2) + x^2 - 6xy + 9y^2 - (4x^2 - 2xy + 6xy - 3y^2) = 3x^2 - 12y^2 + x^2 - 6xy + 9y^2 - 4x^2 - 4xy + 3y^2 = -10xy$

7

解説

- $(x-8)(x+2) + (4-x)(4+x) = x^2 - 6x - 16 + 16 - x^2 = -6x$
 $x = 250$ を代入すると
 $-6 \times 250 = -1500$

8

解説

- $x^2 + y^2 = (x+y)^2 - 2xy = 2^2 - 2 \times \left(-\frac{45}{4}\right) = \frac{53}{2}$ ㊦

9

解説

- $(a+2b-c)^2 = ((a+2b)-c)^2 = (a+2b)^2 - 2(a+2b)c + c^2 = a^2 + 4ab + 4b^2 - 2ac - 4bc + c^2 = a^2 + 4b^2 + c^2 + 4ab - 4bc - 2ca$ ㊦

- 例題 $(a+2b-c)^2 = a^2 + (2b)^2 + (-c)^2 + 2 \times a \times 2b + 2 \times 2b \times (-c) + 2 \times (-c) \times a = a^2 + 4b^2 + c^2 + 4ab - 4bc - 2ca$ ㊦

10

解説

- (1) $(x+y+3)(x+y+2) = ((x+y)+3)((x+y)+2) = (x+y)^2 + 5(x+y) + 6 = x^2 + 2xy + y^2 + 5x + 5y + 6$
 (2) $(2a-3b+1)(2a-3b-4) = ((2a-3b)+1)((2a-3b)-4) = (2a-3b)^2 - 3(2a-3b) - 4 = 4a^2 - 12ab + 9b^2 - 6a + 9b - 4$

11

解説

- (1) $(x^2+x+1)(x^2+x+3) = ((x^2+x)+1)((x^2+x)+3) = (x^2+x)^2 + 4(x^2+x) + 3 = x^4 + 2x^3 + x^2 + 4x^2 + 4x + 3 = x^4 + 2x^3 + 5x^2 + 4x + 3$
 (2) $(x^2+3x+2)(x^2-3x+2) = (x^2+2+3x)((x^2+2)-3x)$

$$= (x^2+2)^2 - (3x)^2 = x^4 + 4x^2 + 4 - 9x^2 = x^4 - 5x^2 + 4$$

- (3) $(a^2+ab+b^2)(a^2-ab+b^2) = ((a^2+b^2)+ab)((a^2+b^2)-ab) = (a^2+b^2)^2 - (ab)^2 = a^4 + 2a^2b^2 + b^4 - a^2b^2 = a^4 + a^2b^2 + b^4$

12

解説

- $(x+1)^2(x-1)^2 = (x+1)(x+1)(x-1)(x-1) = ((x+1)(x-1))^2 = (x^2-1)^2 = (x^2)^2 - 2x^2 + 1 = x^4 - 2x^2 + 1$ ㊦
- 和と差の積の公式
差の平方の公式

13

解説

- (1) $(x-2)(x+2)(x^2+4) = ((x-2)(x+2))(x^2+4) = (x^2-4)(x^2+4) = (x^2)^2 - 4^2 = x^4 - 16$
 (2) $(x-5y)(x^2+25y^2)(x+5y) = (x+5y)(x-5y)(x^2+25y^2) = ((x+5y)(x-5y))(x^2+25y^2) = (x^2-25y^2)(x^2+25y^2) = (x^2)^2 - (25y^2)^2 = x^4 - 625y^4$
 (3) $(a+b)^2(a-b)^2(a^2+b^2)^2 = ((a+b)(a-b)(a^2+b^2))^2 = ((a^2-b^2)(a^2+b^2))^2 = (a^4-b^4)^2 = a^8 - 2a^4b^4 + b^8$

14

解説

- $(x-1)(x+2)(x+4)(x+7) = ((x-1)(x+7))((x+2)(x+4)) = ((x^2+6x)-7)((x^2+6x)+8) = (x^2+6x)^2 + (x^2+6x) - 56 = x^4 + 12x^3 + 36x^2 + x^2 + 6x - 56 = x^4 + 12x^3 + 37x^2 + 6x - 56$

1

解説

- (1) $(x+2)(y+3) = xy + 3x + 2y + 6$
- (2) $(x+7)(y-2) = xy - 2x + 7y - 14$
- (3) $(a-4)(b+8) = ab + 8a - 4b - 32$
- (4) $(k-11)(l-7) = kl - 7k - 11l + 77$
- (5) $(a-2b)(c+3d) = ac + 3ad - 2bc - 6bd$
- (6) $(p+5q)(x-13y) = px - 13py + 5qx - 65qy$
- (7) $(2x+1)(x+3) = 2x^2 + 6x + x + 3 = 2x^2 + 7x + 3$
- (8) $(2a-3)(a+5) = 2a^2 + 10a - 3a - 15 = 2a^2 + 7a - 15$
- (9) $(8n+3)(7n-5) = 56n^2 - 40n + 21n - 15 = 56n^2 - 19n - 15$
- (10) $(4x-y)(5x+2y) = 20x^2 + 8xy - 5xy - 2y^2 = 20x^2 + 3xy - 2y^2$
- (11) $(9m+2n)(4m-3n) = 36mn - 27n^2 + 8m^2 - 6mn = 8m^2 + 30mn - 27n^2$
- (12) $(a+b+c)(a-b) = a^2 - ab + ab - b^2 + ac - bc$
 $= a^2 - b^2 + ac - bc$
- (13) $(x+y+1)(x-2y+3) = x(x-2y+3) + y(x-2y+3) + (x-2y+3)$
 $= x^2 - 2xy + 3x + xy - 2y^2 + 3y + x - 2y + 3$
 $= x^2 - xy + 4x - 2y^2 + y + 3$
- (14) $(x-y)(2-y-x) = x(2-y-x) - y(2-y-x)$
 $= 2x - xy - x^2 - 2y + y^2 + xy$
 $= -x^2 + 2x + y^2 - 2y$

2

解説

- (1) $(x+3)(x+9) = x^2 + (3+9)x + 3 \times 9 = x^2 + 12x + 27$
- (2) $(x-4)(x+8) = x^2 + (-4+8)x + (-4) \times 8 = x^2 + 4x - 32$
- (3) $(t-6)(t-7) = t^2 + (-6+(-7))t + (-6) \times (-7) = t^2 - 13t + 42$
- (4) $(a+12)(a-2) = a^2 + (12+(-2))a + 12 \times (-2) = a^2 + 10a - 24$
- (5) $(x + \frac{1}{2})(x + \frac{3}{2}) = x^2 + (\frac{1}{2} + \frac{3}{2})x + \frac{1}{2} \times \frac{3}{2} = x^2 + 2x + \frac{3}{4}$
- (6) $(x + \frac{1}{4})(x - \frac{3}{4}) = x^2 + (\frac{1}{4} + (-\frac{3}{4}))x + \frac{1}{4} \times (-\frac{3}{4}) = x^2 - \frac{1}{2}x - \frac{3}{16}$
- (7) $(-3+x)(x+6) = (x-3)(x+6) = x^2 + (-3+6)x + (-3) \times 6 = x^2 + 3x - 18$
- (8) $(2x+3)(2x+1) = (2x)^2 + (3+1) \times 2x + 3 \times 1 = 4x^2 + 8x + 3$
- (9) $(4a+3)(4a-7) = (4a)^2 + (3-7) \times 4a + 3 \times (-7) = 16a^2 - 16a - 21$
- (10) $(3m-1)(3m+4) = (3m)^2 + (-1+4) \times 3m + (-1) \times 4 = 9m^2 + 9m - 4$
- (11) $(8y-1)(8y-5) = (8y)^2 + (-1-5) \times 8y + (-1) \times (-5) = 64y^2 - 48y + 5$
- (12) $(5x+2)(5x-6) = (5x)^2 + (2-6) \times 5x + 2 \times (-6) = 25x^2 - 20x - 12$
- (13) $(7x-3)(7x+9) = (7x)^2 + (-3+9) \times 7x + (-3) \times 9 = 49x^2 + 42x - 27$

3

解説

- (1) $(x+5)^2 = x^2 + 2 \times 5 \times x + 5^2 = x^2 + 10x + 25$
- (2) $(x-6)^2 = x^2 - 2 \times 6 \times x + 6^2 = x^2 - 12x + 36$
- (3) $(a+10)^2 = a^2 + 2 \times 10 \times a + 10^2 = a^2 + 20a + 100$
- (4) $(x-3)^2 = x^2 - 2 \times 3 \times x + 3^2 = x^2 - 6x + 9$
- (5) $(x-7)^2 = x^2 - 2 \times 7 \times x + 7^2 = x^2 - 14x + 49$

(6) $(y-12)^2 = y^2 - 2 \times 12 \times y + 12^2 = y^2 - 24y + 144$

(7) $(2x+5)^2 = (2x)^2 + 2 \times 5 \times 2x + 5^2 = 4x^2 + 20x + 25$

(8) $(3a+2)^2 = (3a)^2 + 2 \times 2 \times 3a + 2^2 = 9a^2 + 12a + 4$

(9) $(5p-8)^2 = (5p)^2 - 2 \times 8 \times 5p + 8^2 = 25p^2 - 80p + 64$

(10) $(3x+2y)^2 = (3x)^2 + 2 \times 2y \times 3x + (2y)^2 = 9x^2 + 12xy + 4y^2$

(11) $(-a+4b)^2 = (-a)^2 + 2 \times 4b \times (-a) + (4b)^2 = a^2 - 8ab + 16b^2$

(12) $(7x-2y)^2 = (7x)^2 - 2 \times 2y \times 7x + (2y)^2 = 49x^2 - 28xy + 4y^2$

(13) $(x + \frac{1}{3})^2 = x^2 + 2 \times \frac{1}{3} \times x + (\frac{1}{3})^2 = x^2 + \frac{2}{3}x + \frac{1}{9}$

(14) $(x - \frac{1}{2})^2 = x^2 - 2 \times \frac{1}{2} \times x + (\frac{1}{2})^2 = x^2 - x + \frac{1}{4}$

(15) $(x - \frac{1}{4})^2 = x^2 - 2 \times \frac{1}{4} \times x + (\frac{1}{4})^2 = x^2 - \frac{1}{2}x + \frac{1}{16}$

4

解説

- (1) $(x+2)(x-2) = x^2 - 2^2 = x^2 - 4$
- (2) $(a+9)(a-9) = a^2 - 9^2 = a^2 - 81$
- (3) $(x-6)(x+6) = x^2 - 6^2 = x^2 - 36$
- (4) $(x-12)(x+12) = x^2 - 12^2 = x^2 - 144$
- (5) $(5x+3y)(5x-3y) = (5x)^2 - (3y)^2 = 25x^2 - 9y^2$
- (6) $(4a-9b)(4a+9b) = (4a)^2 - (9b)^2 = 16a^2 - 81b^2$
- (7) $(x + \frac{1}{3})(x - \frac{1}{3}) = x^2 - (\frac{1}{3})^2 = x^2 - \frac{1}{9}$
- (8) $(\frac{1}{4}a - \frac{1}{5}b)(\frac{1}{4}a + \frac{1}{5}b) = (\frac{1}{4}a)^2 - (\frac{1}{5}b)^2 = \frac{1}{16}a^2 - \frac{1}{25}b^2$
- (9) $(\frac{1}{7}b + a)(a - \frac{1}{7}b) = (a + \frac{1}{7}b)(a - \frac{1}{7}b) = a^2 - (\frac{1}{7}b)^2 = a^2 - \frac{1}{49}b^2$

5

解説

- (1) $(x-4)(x-5) = x^2 - 9x + 20$
- (2) $(x+6)^2 = x^2 + 12x + 36$
- (3) $(a+7)(a-7) = a^2 - 49$
- (4) $(x-2)(x+5) = x^2 + 3x - 10$
- (5) $(x-2)^2 = x^2 - 4x + 4$
- (6) $(3x+4)(3x-4) = 9x^2 - 16$
- (7) $(a-4b)^2 = a^2 - 8ab + 16b^2$
- (8) $(4a-5b)(4a+5b) = 16a^2 - 25b^2$
- (9) $(x+2)(x+7) = x^2 + 9x + 14$
- (10) $(2x+5y)^2 = 4x^2 + 20xy + 25y^2$
- (11) $(3x+5)(3x-5) = 9x^2 - 25$
- (12) $(5x-2y)^2 = 25x^2 - 20xy + 4y^2$
- (13) $(2a+5b)(2a-5b) = 4a^2 - 25b^2$
- (14) $(x+6)(x-4) = x^2 + 2x - 24$
- (15) $(x+4)(x-4) = x^2 - 16$
- (16) $(a+3)(a-8) = a^2 - 5a - 24$
- (17) $(2a-3)^2 = 4a^2 - 12a + 9$
- (18) $(3x-4y)^2 = 9x^2 - 24xy + 16y^2$
- (19) $(2x-3)(2x+3) = 4x^2 - 9$
- (20) $(x+5)(x-5) = x^2 - 25$
- (21) $(y-2)(y-4) = y^2 - 6y + 8$
- (22) $(x-4)(x+2) = x^2 - 2x - 8$
- (23) $(3x+5y)^2 = 9x^2 + 30xy + 25y^2$
- (24) $(2a+b)(2a-b) = 4a^2 - b^2$
- (25) $(p-7)(p+6) = p^2 - p - 42$
- (26) $(a-4b)(a-7b) = a^2 - 11ab + 28b^2$
- (27) $(2a+3x)(2a-3x) = 4a^2 - 9x^2$
- (28) $(5x-2y)^2 = 25x^2 - 20xy + 4y^2$
- (29) $(x-5y)(x+2y) = x^2 - 3xy - 10y^2$
- (30) $(3a-2b)^2 = 9a^2 - 12ab + 4b^2$

6

解説

- (1) $(x+6)^2 - (x+12)(x+3) = (x^2 + 12x + 36) - (x^2 + 15x + 36)$
 $= x^2 + 12x + 36 - x^2 - 15x - 36$
 $= -3x$
- (2) $(x-3)(x+15) - (x-6)^2 = (x^2 + 12x - 45) - (x^2 - 12x + 36)$
 $= x^2 + 12x - 45 - x^2 + 12x - 36$
 $= 24x - 81$
- (3) $(2x+3)^2 - (2x+5)(2x-5) = (4x^2 + 12x + 9) - (4x^2 - 25)$
 $= 4x^2 + 12x + 9 - 4x^2 + 25$
 $= 12x + 34$
- (4) $(2a-3)(5a+2) + (a+5)(a+6) = (10a^2 - 11a - 6) + (a^2 + 11a + 30)$
 $= 10a^2 - 11a - 6 + a^2 + 11a + 30$
 $= 11a^2 + 24$
- (5) $(2b-a)^2 - (a+5b)(a-b) = (4b^2 - 4ab + a^2) - (a^2 + 4ab - 5b^2)$
 $= 4b^2 - 4ab + a^2 - a^2 - 4ab + 5b^2$
 $= 9b^2 - 8ab$
- (6) $(6x+5y)(3x-4y) + (x+4y)(2x+y) = (18x^2 - 9xy - 20y^2) + (2x^2 + 9xy + 4y^2)$
 $= 18x^2 - 9xy - 20y^2 + 2x^2 + 9xy + 4y^2$
 $= 20x^2 - 16y^2$
- (7) $(3x-2y)(2y+3x) + (5x-2y)^2 = (3x-2y)(3x+2y) + (5x-2y)^2$
 $= (9x^2 - 4y^2) + (25x^2 - 20xy + 4y^2)$
 $= 9x^2 - 4y^2 + 25x^2 - 20xy + 4y^2$
 $= 34x^2 - 20xy$
- (8) $(2a-6b)^2 + (4a+3b)^2 = (4a^2 - 24ab + 36b^2) + (16a^2 + 24ab + 9b^2)$
 $= 4a^2 - 24ab + 36b^2 + 16a^2 + 24ab + 9b^2$
 $= 20a^2 + 45b^2$
- (9) $3(a+4)(a-5) - 2(a-1)(a+2) = 3(a^2 - a - 20) - 2(a^2 + a - 2)$
 $= 3a^2 - 3a - 60 - 2a^2 - 2a + 4$
 $= a^2 - 5a - 56$

7

解説

$(2a+3)^2 - 4a(a+5) = 4a^2 + 12a + 9 - 4a^2 - 20a = -8a + 9$

この式に $a = -\frac{1}{8}$ を代入すると、求める式の値は

$$-8 \times \left(-\frac{1}{8}\right) + 9 = 1 + 9 = 10$$

8

解説

$x^2 + y^2 = (x+y)^2 - 2xy$

$$= \left(\frac{17}{2}\right)^2 - 2 \times 18 = \frac{289}{4} - 36 = \frac{145}{4}$$

9

解説

$$(1) (a-b+1)^2 = [(a-b)+1]^2 \\ = (a-b)^2 + 2(a-b) \times 1 + 1^2 \\ = a^2 - 2ab + b^2 + 2a - 2b + 1$$

$$(2) (2a-b-c)^2 = [(2a-b)-c]^2 \\ = (2a-b)^2 - 2(2a-b)c + c^2 \\ = 4a^2 - 4ab + b^2 - 4ac + 2bc + c^2 \\ = 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac$$

$$(3) (a+2b+3c)^2 = [(a+2b)+3c]^2 \\ = (a+2b)^2 + 2(a+2b) \times 3c + (3c)^2 \\ = a^2 + 4ab + 4b^2 + 6ac + 12bc + 9c^2 \\ = a^2 + 4b^2 + 9c^2 + 4ab + 12bc + 6ca$$

$$(4) (3x+5y+4z)^2 = [(3x+5y)+4z]^2 \\ = (3x+5y)^2 + 2(3x+5y) \times 4z + (4z)^2 \\ = 9x^2 + 30xy + 25y^2 + 24xz + 40yz + 16z^2 \\ = 9x^2 + 25y^2 + 16z^2 + 30xy + 40yz + 24xz$$

$$(5) (9x+8y-6z)^2 = [(9x+8y)-6z]^2 \\ = (9x+8y)^2 - 2(9x+8y) \times 6z + (6z)^2 \\ = 81x^2 + 144xy + 64y^2 - 108xz - 96yz + 36z^2 \\ = 81x^2 + 64y^2 + 36z^2 + 144xy - 96yz - 108xz$$

$$(6) (-7x+5y-3z)^2 = [(-7x+5y)-3z]^2 \\ = (-7x+5y)^2 - 2(-7x+5y) \times 3z + (3z)^2 \\ = 49x^2 - 70xy + 25y^2 + 42xz - 30yz + 9z^2 \\ = 49x^2 + 25y^2 + 9z^2 - 70xy - 30yz + 42xz$$

例解 公式 $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ を使って展開すると、(1)、(2)は次のようになる。

$$(1) (a-b+1)^2 = a^2 + (-b)^2 + 1^2 + 2 \times a \times (-b) + 2 \times (-b) \times 1 + 2 \times 1 \times a \\ = a^2 - 2ab + b^2 + 2a - 2b + 1$$

$$(2) (2a-b-c)^2 = (2a)^2 + (-b)^2 + (-c)^2 + 2 \times 2a \times (-b) + 2 \times (-b) \times (-c) + 2 \times (-c) \times 2a \\ = 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ca$$

(3)~(6)も同様に公式を使って展開できる。

10

解説

$$(1) (x+y+8)(x+y+7) = [(x+y)+8][(x+y)+7] \\ = (x+y)^2 + 15(x+y) + 56 \\ = x^2 + 2xy + y^2 + 15x + 15y + 56$$

$$(2) (a+b+3)(a+b-4) = [(a+b)+3][(a+b)-4] \\ = (a+b)^2 - (a+b) - 12 \\ = a^2 + 2ab + b^2 - a - b - 12$$

$$(3) (x-2y+3)(x-2y-9) = [(x-2y)+3][(x-2y)-9] \\ = (x-2y)^2 - 6(x-2y) - 27 \\ = x^2 - 4xy + 4y^2 - 6x + 12y - 27$$

$$(4) (3a-2b+5)(3a-2b-3) = [(3a-2b)+5][(3a-2b)-3]$$

$$= (3a-2b)^2 + 2(3a-2b) - 15 \\ = 9a^2 - 12ab + 4b^2 + 6a - 4b - 15$$

$$(5) (2x+3y-z)(2x+3y+z) = [(2x+3y)-z][(2x+3y)+z] \\ = (2x+3y)^2 - z^2 \\ = 4x^2 + 12xy + 9y^2 - z^2$$

$$(6) (4p-5q+2r)(4p-5q-3r) = [(4p-5q)+2r][(4p-5q)-3r] \\ = (4p-5q)^2 - r(4p-5q) - 6r^2 \\ = 16p^2 - 40pq + 25q^2 - 4pr + 5qr - 6r^2 \\ = 16p^2 + 25q^2 - 6r^2 - 40pq + 5qr - 4rp$$

11

解説

$$(1) (a^2+3a+4)(a^2+3a-7) = [(a^2+3a)+4][(a^2+3a)-7] \\ = (a^2+3a)^2 - 3(a^2+3a) - 28 \\ = a^4 + 6a^3 + 9a^2 - 3a^2 - 9a - 28 \\ = a^4 + 6a^3 + 6a^2 - 9a - 28$$

$$(2) (x^2+6x+1)(x^2+6x-1) = [(x^2+6x)+1][(x^2+6x)-1] \\ = (x^2+6x)^2 - 1^2 \\ = x^4 + 12x^3 + 36x^2 - 1$$

$$(3) (x^2-3xy+y^2)(x^2+3xy+y^2) = [(x^2+y^2)-3xy][(x^2+y^2)+3xy] \\ = (x^2+y^2)^2 - (3xy)^2 \\ = x^4 + 2x^2y^2 + y^4 - 9x^2y^2 \\ = x^4 - 7x^2y^2 + y^4$$

$$(4) (a^2+3ab-2b^2)(a^2-3ab-2b^2) = [(a^2-2b^2)+3ab][(a^2-2b^2)-3ab] \\ = (a^2-2b^2)^2 - (3ab)^2 \\ = a^4 - 4a^2b^2 + 4b^4 - 9a^2b^2 \\ = a^4 - 13a^2b^2 + 4b^4$$

12

解説

$$(1) (a-5)^2(a+5)^2 = [(a-5)(a+5)]^2 \\ = (a^2-25)^2 = a^4 - 50a^2 + 625$$

$$(2) (x+4)^2(x-4)^2 = [(x+4)(x-4)]^2 \\ = (x^2-16)^2 = x^4 - 32x^2 + 256$$

$$(3) (p+3q)^2(p-3q)^2 = [(p+3q)(p-3q)]^2 \\ = (p^2-9q^2)^2 = p^4 - 18p^2q^2 + 81q^4$$

$$(4) (2x-y)^2(2x+y)^2 = [(2x-y)(2x+y)]^2 \\ = (4x^2-y^2)^2 = 16x^4 - 8x^2y^2 + y^4$$

$$(5) (3a-4b)^2(3a+4b)^2 = [(3a-4b)(3a+4b)]^2 = (9a^2-16b^2)^2 \\ = 81a^4 - 288a^2b^2 + 256b^4$$

$$(6) (7y+2x)^2(2x-7y)^2 = [(2x+7y)(2x-7y)]^2 = (4x^2-49y^2)^2 \\ = 16x^4 - 392x^2y^2 + 2401y^4$$

13

解説

$$(1) (x-4)(x+4)(x^2+16) = [(x-4)(x+4)](x^2+16) \\ = (x^2-16)(x^2+16)$$

$$= (x^2)^2 - 16^2$$

$$= x^4 - 256$$

$$(2) (3x-y)(9x^2+y^2)(3x+y) = (3x+y)(3x-y)(9x^2+y^2) \\ = [(3x+y)(3x-y)](9x^2+y^2) \\ = (9x^2-y^2)(9x^2+y^2) \\ = (9x^2)^2 - (y^2)^2 \\ = 81x^4 - y^4$$

$$(3) (x-y)^2(x+y)^2(x^2+y^2)^2 = [(x-y)(x+y)]^2(x^2+y^2)^2 \\ = (x^2-y^2)^2(x^2+y^2)^2 \\ = [(x^2-y^2)(x^2+y^2)]^2 \\ = (x^4-y^4)^2 \\ = x^8 - 2x^4y^4 + y^8$$

$$(4) (2a-3b)^2(2a+3b)^2(4a^2+9b^2)^2 = [(2a-3b)(2a+3b)]^2(4a^2+9b^2)^2 \\ = (4a^2-9b^2)^2(4a^2+9b^2)^2 \\ = [(4a^2-9b^2)(4a^2+9b^2)]^2 \\ = [(4a^2)^2 - (9b^2)^2]^2 \\ = (16a^4 - 81b^4)^2 \\ = 256a^8 - 2592a^4b^4 + 6561b^8$$

14

解説

$$(1) (x-1)(x-2)(x-3)(x-4) = (x-1)(x-4) \times (x-2)(x-3) \\ = (x^2-5x+4)(x^2-5x+6) \\ = [(x^2-5x)+4][(x^2-5x)+6] \\ = (x^2-5x)^2 + 10(x^2-5x) + 24 \\ = x^4 - 10x^3 + 25x^2 + 10x^2 - 50x + 24 \\ = x^4 - 10x^3 + 35x^2 - 50x + 24$$

$$(2) (x-1)(x-2)(x+3)(x+4) = [(x-1)(x+3)][(x-2)(x+4)] \\ = [(x^2+2x)-3][(x^2+2x)-8] \\ = (x^2+2x)^2 - 11(x^2+2x) + 24 \\ = x^4 + 4x^3 + 4x^2 - 11x^2 - 22x + 24 \\ = x^4 + 4x^3 - 7x^2 - 22x + 24$$

$$(3) (x-2)(x-3)(x+4)(x+5) = (x-2)(x+4) \times (x-3)(x+5) \\ = (x^2+2x-8)(x^2+2x-15) \\ = [(x^2+2x)-8][(x^2+2x)-15] \\ = (x^2+2x)^2 - 23(x^2+2x) + 120 \\ = x^4 + 4x^3 + 4x^2 - 23x^2 - 46x + 120 \\ = x^4 + 4x^3 - 19x^2 - 46x + 120$$

$$(4) (x+1)(x+3)(x-5)(x-7) = [(x+1)(x-5)][(x+3)(x-7)] \\ = (x^2-4x-5)(x^2-4x-21) \\ = [(x^2-4x)-5][(x^2-4x)-21] \\ = (x^2-4x)^2 - 26(x^2-4x) + 105 \\ = x^4 - 8x^3 + 16x^2 - 26x^2 + 104x + 105 \\ = x^4 - 8x^3 - 10x^2 + 104x + 105$$

1

解説

- (1) $(3a+2)^2 = (3a)^2 + 2 \cdot 3a \cdot 2 + 2^2$
 $= 9a^2 + 12a + 4$
- (2) $(5x-2y)^2 = (5x)^2 - 2 \cdot 5x \cdot 2y + (2y)^2$
 $= 25x^2 - 20xy + 4y^2$
- (3) $(4x+3)(4x-3) = (4x)^2 - 3^2$
 $= 16x^2 - 9$
- (4) $(-2b-a)(a-2b) = -(a+2b)(a-2b) = -(a^2 - (2b)^2)$
 $= -(a^2 - 4b^2)$
 $= -a^2 + 4b^2$
- (5) $(x+6)(x+7) = x^2 + (6+7)x + 6 \cdot 7$
 $= x^2 + 13x + 42$
- (6) $(2t-3)(2t-5) = (2t)^2 + \{(-3) + (-5)\} \cdot 2t + (-3)(-5)$
 $= 4t^2 - 16t + 15$
- (7) $(4x+1)(3x-2) = 4 \cdot 3x^2 + \{4 \cdot (-2) + 1 \cdot 3\}x + 1 \cdot (-2)$
 $= 12x^2 - 5x - 2$
- (8) $(2a+3b)(3a+5b) = 2 \cdot 3a^2 + \{2 \cdot (5b) + (3b) \cdot 3\}a + 3b \cdot 5b$
 $= 6a^2 + 19ab + 15b^2$
- (9) $(7x-3)(-2x+3) = 7 \cdot (-2)x^2 + \{7 \cdot 3 + (-3) \cdot (-2)\}x + (-3) \cdot 3$
 $= -14x^2 + 27x - 9$

2

解説

- (1) 与式 $= (3x)^2 + 2 \cdot 3x \cdot 5 + 5^2 = 9x^2 + 30x + 25$
- (2) 与式 $= (2p)^2 - 2 \cdot 2p \cdot 5 + 5^2 = 4p^2 - 20p + 25$
- (3) 与式 $= (4a)^2 - 2 \cdot 4a \cdot 3b + (3b)^2 = 16a^2 - 24ab + 9b^2$
- (4) 与式 $= (-x)^2 - 2 \cdot (-x)^2 \cdot x + x^2 = x^4 + 2x^3 + x^2$
- (5) 与式 $= (3a)^2 - 2 \cdot 3a \cdot \frac{1}{2}b + \left(\frac{1}{2}b\right)^2 = 9a^2 - 3ab + \frac{1}{4}b^2$
- (6) 与式 $= (3x)^2 - 5^2 = 9x^2 - 25$
- (7) 与式 $= (2a)^2 - (7b)^2 = 4a^2 - 49b^2$
- (8) 与式 $= (a^2)^2 - (b^2)^2 = a^4 - b^4$
- (9) 与式 $= (-ab)^2 - c^2 = a^2b^2 - c^2$

3

解説

- (1) $(x+2)(x+\textcircled{1}) = x^2 + 6x + \textcircled{2}$
 $2 + \textcircled{1} = 6, 2 \times \textcircled{1} = \textcircled{2}$
 よって $\textcircled{1} = 4, \textcircled{2} = 8$
- (2) $(x-\textcircled{1})(x+5) = x^2 - \textcircled{2}x - 35$
 $-\textcircled{1} + 5 = -\textcircled{2}, -\textcircled{1} \times 5 = -35$
 よって $\textcircled{1} = 7, \textcircled{2} = 2$
- (3) $(x+\textcircled{1})^2 = x^2 + 12x + \textcircled{2}$
 $2 \times \textcircled{1} = 12, \textcircled{1}^2 = \textcircled{2}$
 よって $\textcircled{1} = 6, \textcircled{2} = 36$
- (4) $(\textcircled{1}x-3)^2 = \textcircled{2}x^2 - 30x + 9$

$$\textcircled{1}^2 = \textcircled{2}, 2 \times \textcircled{1} \times 3 = 30$$

- よって $\textcircled{1} = 5, \textcircled{2} = 25$
- (5) $(3x+\textcircled{1})(3x-8) = 9x^2 - \textcircled{2}$
 $\textcircled{1} = 8, \textcircled{2} = 64$
- (6) $(7x-2)(\textcircled{1}x+2) = \textcircled{2}x^2 - 4$
 $\textcircled{1} = 7, \textcircled{2} = 49$

4

解説

- (1) $(ab-6)(ab+3) = a^2b^2 - 3ab - 18$ (2) $\left(x-\frac{1}{2}y\right)\left(x+\frac{1}{2}y\right) = x^2 - \frac{1}{4}y^2$
- (3) $(-2a+5b)^2 = 4a^2 - 20ab + 25b^2$ (4) $(2a+3)(2a+7) = 4a^2 + 20a + 21$
- (5) $(-ab+c)(-ab-c) = a^2b^2 - c^2$ (6) $(7y-2)(7y-6) = 49y^2 - 56y + 12$
- (7) $(a^2+2b)^2 = a^4 + 4a^2b + 4b^2$ (8) $(4x-y)(y+4x) = 16x^2 - y^2$
- (9) $(a^2-3)(a^2+7) = a^4 + 4a^2 - 21$ (10) $(a^2+b^2)(a^2-b^2) = a^4 - b^4$
- (11) $(2t-3)(2t-5) = 4t^2 - 16t + 15$ (12) $(4x-3)(4x+9) = 16x^2 + 24x - 27$
- (13) $(2xy-3)^2 = 4x^2y^2 - 12xy + 9$ (14) $\left(\frac{1}{4}a - \frac{1}{5}b\right)\left(\frac{1}{4}a + \frac{1}{5}b\right) = \frac{1}{16}a^2 - \frac{1}{25}b^2$
- (15) $\left(\frac{1}{7}b + a\right)\left(a - \frac{1}{7}b\right) = a^2 - \frac{1}{49}b^2$ (16) $(3a+1)(3a+5)9a^2 + 18a + 5$
- (17) $(4x-1)(4x+5) = 16x^2 + 16x - 5$ (18) $(x^2+x)^2 = x^4 + 2x^3 + x^2$
- (19) $(-x^2-x)^2 = x^4 + 2x^3 + x^2$ (20) $(ab+3)(ab-4) = a^2b^2 - ab - 12$
- (21) $(3a-4b)(4b+3a) = 9a^2 - 16b^2$ (22) $(xy+ab)(xy+7ab) = x^2y^2 + 8abxy + 7a^2b^2$
- (23) $\left(a + \frac{1}{2}b\right)^2 = a^2 + ab + \frac{1}{4}b^2$ (24) $(4a+3b)(4a-7b) = 16a^2 - 16ab - 21b^2$
- (25) $\left(x - \frac{3}{5}\right)\left(x - \frac{5}{6}\right) = x^2 - \frac{43}{30}x + \frac{1}{2}$ (26) $(5x+6)(5x-8) = 25x^2 - 10x - 48$
- (27) $(x^3+4x)^2 = x^6 + 8x^4 + 16x^2$ (28) $(8y-1)(8y-5) = 64y^2 - 48y + 5$
- (29) $(2x^4-5y^5)^2 = 4x^8 - 20x^4y^5 + 25y^{10}$ (30) $(a^3+b^3)(a^3-b^3) = a^6 - b^6$

5

解説

- (1) $(x+2)(x-8) + (x+3)^2 = (x^2 - 6x - 16) + (x^2 + 6x + 9)$
 $= x^2 - 6x - 16 + x^2 + 6x + 9$
 $= 2x^2 - 7$
- (2) $(x-3)(x+5) - (x+6)(x-7) = (x^2 + 2x - 15) - (x^2 - x - 42)$
 $= x^2 + 2x - 15 - x^2 + x + 42$
 $= 3x + 27$
- (3) $(3x-2)^2 - (x+1)(x-4) = (9x^2 - 12x + 4) - (x^2 - 3x - 4)$
 $= 9x^2 - 12x + 4 - x^2 + 3x + 4$
 $= 8x^2 - 9x + 8$
- (4) $(2m+3n)^2 + (2m-3n)^2 = (4m^2 + 12mn + 9n^2) + (4m^2 - 12mn + 9n^2)$
 $= 4m^2 + 12mn + 9n^2 + 4m^2 - 12mn + 9n^2$
 $= 8m^2 + 18n^2$
- (5) $(x+2y)(x-y) - (3x-y)^2 = (x^2 + xy - 2y^2) - (9x^2 - 6xy + y^2)$
 $= x^2 + xy - 2y^2 - 9x^2 + 6xy - y^2$
 $= -8x^2 + 7xy - 3y^2$
- (6) $(-a+3)(-a-3) + 2(a+1)^2 = (a^2 - 9) + 2(a^2 + 2a + 1)$

$$= a^2 - 9 + 2a^2 + 4a + 2$$

$$= 3a^2 + 4a - 7$$

- (7) $(x+3)(x-7) + (x-3)^2 - (2x-3)(x-5)$
 $= (x^2 - 4x - 21) + (x^2 - 6x + 9) - (2x^2 - 13x + 15)$
 $= x^2 - 4x - 21 + x^2 - 6x + 9 - 2x^2 + 13x - 15$
 $= 3x - 27$
- (8) $(5a+3b)(5a-3b) - (2a-7b)^2 - (a-4b)(a+2b)$
 $= (25a^2 - 9b^2) - (4a^2 - 28ab + 49b^2) - (a^2 - 2ab - 8b^2)$
 $= 25a^2 - 9b^2 - 4a^2 + 28ab - 49b^2 - a^2 + 2ab + 8b^2$
 $= 20a^2 + 30ab - 50b^2$

6

解説

- (1) $(x+2y+1)(x+2y-1) = ((x+2y)+1)((x+2y)-1)$
 $= (x+2y)^2 - 1^2$
 $= x^2 + 4xy + 4y^2 - 1$
- (2) $(a-3b+c)(a-3b-c) = ((a-3b)+c)((a-3b)-c)$
 $= (a-3b)^2 - c^2$
 $= a^2 - 6ab + 9b^2 - c^2$
- (3) $(x^2+6x+2)(x^2+6x-2) = ((x^2+6x)+2)((x^2+6x)-2)$
 $= (x^2+6x)^2 - 2^2$
 $= x^4 + 12x^3 + 36x^2 - 4$
- (4) $(x-3xy+y)(x+3xy+y) = ((x+y)-3xy)((x+y)+3xy)$
 $= (x+y)^2 - (3xy)^2$
 $= x^2 + 2xy + y^2 - 9x^2y^2$
- (5) $(a^2+3ab-2b^2)(a^2-3ab-2b^2) = ((a^2-2b^2)+3ab)((a^2-2b^2)-3ab)$
 $= (a^2-2b^2)^2 - (3ab)^2$
 $= a^4 - 4a^2b^2 + 4b^4 - 9a^2b^2$
 $= a^4 - 13a^2b^2 + 4b^4$

7

解説

- (1) $(x^2-3x+1)(x^2+3x+1) = ((x^2+1)-3x)((x^2+1)+3x)$
 $= (x^2+1)^2 - 9x^2$
 $= x^4 - 7x^2 + 1$
- (2) (与式) $= ((x^2+2)+x)((x^2+2)-x) = (x^2+2)^2 - x^2 = x^4 + 3x^2 + 4$
- (3) $(x^2+3x+2)(x^2-3x+2) = ((x^2+2)+3x)((x^2+2)-3x) = (x^2+2)^2 - (3x)^2$
 $= x^4 - 5x^2 + 4$
- (4) (与式) $= ((a^2+9)+3a)((a^2+9)-3a) = (a^2+9)^2 - (3a)^2$
 $= a^4 + 18a^2 + 81 - 9a^2 = a^4 + 9a^2 + 81$
- (5) $(x^2-2xy+4y^2)(x^2+2xy+4y^2) = (x^2+4y^2)^2 - (2xy)^2$
 $= x^4 + 8x^2y^2 + 16y^4 - 4x^2y^2$
 $= x^4 + 4x^2y^2 + 16y^4$

8

解説

$$(1) \text{ (与式)} = \{(x^2 + 4x) + 1\} \{(x^2 + 4x) + 3\} = (x^2 + 4x)^2 + (1 + 3)(x^2 + 4x) + 1 \cdot 3$$

$$= (x^2)^2 + 2 \cdot x^2 \cdot 4 + (4x)^2 + 4 \cdot x^2 + 4 \cdot 4x + 3 = x^4 + 8x^2 + 16x^2 + 4x^2 + 16x + 3$$

$$= x^4 + 8x^2 + 20x^2 + 16x + 3$$

$$(2) \text{ (与式)} = \{(x^2 - 1) + 2x\} \{(x^2 - 1) + 3x\} = (x^2 - 1)^2 + (2x + 3x)(x^2 - 1) + 2x \cdot 3x$$

$$= (x^2)^2 - 2 \cdot x^2 \cdot 1 + 1^2 + 5x(x^2 - 1) + 6x^2 = x^4 - 2x^2 + 1 + 5x^3 - 5x + 6x^2$$

$$= x^4 + 5x^3 + 4x^2 - 5x + 1$$

$$(3) \text{ (与式)} = \{x + (2y - 3z)\} \{x - (2y - 3z)\} = x^2 - (2y - 3z)^2$$

$$= x^2 - \{(2y)^2 - 2 \cdot 2y \cdot 3z + (3z)^2\} = x^2 - 4y^2 + 12yz - 9z^2$$

$$(4) \text{ (与式)} = \{a^2 - (ab - b^2)\} \{a^2 + (ab - b^2)\} = (a^2)^2 - (ab - b^2)^2$$

$$= a^4 - \{(ab)^2 - 2 \cdot ab \cdot b^2 + (b^2)^2\} = a^4 - a^2b^2 + 2ab^3 - b^4$$

$$(5) \text{ (与式)} = \{x^2 + (2 + 3)x + 2 \cdot 3\} \{(x^2 + 5x)\} = \{(x^2 + 5x) + 6\} \{(x^2 + 5x)\}$$

$$= (x^2 + 5x)^2 + 6(x^2 + 5x) = (x^2)^2 + 2 \cdot x^2 \cdot 5x + (5x)^2 + 6x^2 + 30x$$

$$= x^4 + 10x^3 + 31x^2 + 30x$$

$$(6) \text{ (与式)} = (x^2 - 6)\{x^2 + (2 - 3)x + 2 \cdot (-3)\} = (x^2 - 6)(x^2 - x - 6)$$

$$= (x^2 - 6)\{(x^2 - 6) - x\} = (x^2 - 6)^2 - (x^2 - 6)x$$

$$= (x^2)^2 - 2 \cdot x^2 \cdot 6 + 6^2 - x^3 + 6x = x^4 - x^3 - 12x^2 + 6x + 36$$

$$(7) \text{ (与式)} = (x^2 - 4)(x^2 - 9) = (x^2)^2 - (4 + 9)x^2 + (-4) \cdot (-9)$$

$$= x^4 - 13x^2 + 36$$

$$(8) \text{ (与式)} = (x - 1)(x + 1) \times (x - 3)(x + 3) = (x^2 - 1)(x^2 - 9) = x^4 - 10x^2 + 9$$

$$(9) \text{ (与式)} = (x + 1)(x - 4) \times (x - 1)(x - 2) = (x^2 - 3x - 4)(x^2 - 3x + 2)$$

$$= \{(x^2 - 3x) - 4\} \{(x^2 - 3x) + 2\} = (x^2 - 3x)^2 - 2(x^2 - 3x) - 8$$

$$= x^4 - 6x^3 + 9x^2 - 2x^2 + 6x - 8 = x^4 - 6x^3 + 7x^2 + 6x - 8$$

$$(10) \text{ (与式)} = (x + 1)(x + 6) \times (x + 2)(x + 3) = (x^2 + 7x + 6)(x^2 + 5x + 6)$$

$$= \{(x^2 + 6) + 7x\} \{(x^2 + 6) + 5x\} = (x^2 + 6)^2 + 12x(x^2 + 6) + 35x^2$$

$$= x^4 + 12x^2 + 36 + 12x^3 + 72x + 35x^2 = x^4 + 12x^3 + 47x^2 + 72x + 36$$

$$(11) \text{ (与式)} = \{(x - 2)(x + 2)(x^2 + 4)\}^2 = \{(x^2 - 4)(x^2 + 4)\}^2 = (x^4 - 16)^2 = x^8 - 32x^4 + 256$$

$$(12) \text{ (与式)} = (x^2 - a^2)(x^2 + a^2)(x^4 + a^4) = (x^4 - a^4)(x^4 + a^4) = x^8 - a^8$$

9

解説

$$(1) (2x - y)^2 + (x + 2y)(x - 2y) - (x - y)(5x + 3y)$$

$$= (4x^2 - 4xy + y^2) + (x^2 - 4y^2) - (5x^2 - 2xy - 3y^2)$$

$$= 4x^2 - 4xy + y^2 + x^2 - 4y^2 - 5x^2 + 2xy + 3y^2$$

$$= -2xy$$

$$(2) (a + 2)^2 - (a + 2)(b + 2) + a(b - 2)$$

$$= (a^2 + 4a + 4) - (ab + 2a + 2b + 4) + (ab - 2a)$$

$$= a^2 + 4a + 4 - ab - 2a - 2b - 4 + ab - 2a$$

$$= a^2 - 2b$$

$$(3) (2a - b)(a + 2b) - (a + b)^2 + 3b^2$$

$$= (2a^2 + 3ab - 2b^2) - (a^2 + 2ab + b^2) + 3b^2$$

$$= 2a^2 + 3ab - 2b^2 - a^2 - 2ab - b^2 + 3b^2$$

$$= a^2 + ab$$

$$(4) (a + b + c)^2 - (a - b + c)^2 = \{(a + c) + b\}^2 - \{(a + c) - b\}^2$$

$$= \{(a + c)^2 + 2b(a + c) + b^2\} - \{(a + c)^2 - 2b(a + c) + b^2\}$$

$$= (a + c)^2 + 2b(a + c) + b^2 - (a + c)^2 + 2b(a + c) - b^2$$

$$= 4b(a + c)$$

$$= 4ab + 4bc$$

$$(5) (x + y + 1)(x + y - 1) - (x - y)^2 = \{(x + y) + 1\} \{(x + y) - 1\} - (x - y)^2$$

$$= \{(x + y)^2 - 1\} - (x - y)^2$$

$$= (x + y)^2 - 1 - (x - y)^2$$

$$= (x^2 + 2xy + y^2) - 1 - (x^2 - 2xy + y^2)$$

$$= x^2 + 2xy + y^2 - 1 - x^2 + 2xy - y^2$$

$$= 4xy - 1$$

$$(6) (2x - 2y - 1)^2 - 4(x - y)^2 = \{2(x - y) - 1\}^2 - 4(x - y)^2$$

$$= \{4(x - y)^2 - 4(x - y) + 1\} - 4(x - y)^2$$

$$= 4(x - y)^2 - 4(x - y) + 1 - 4(x - y)^2$$

$$= -4(x - y) + 1$$

$$= -4x + 4y + 1$$

$$(7) (a + b)^2 - (a - 1 + b)^2 = (a + b)^2 - \{(a + b) - 1\}^2$$

$$= (a + b)^2 - \{(a + b)^2 - 2(a + b) + 1\}$$

$$= (a + b)^2 - (a + b)^2 + 2(a + b) - 1$$

$$= 2(a + b) - 1$$

$$= 2a + 2b - 1$$

10

解説

$$(1) (2x - y)^2 + (x + y)(x + 3y) = (4x^2 - 4xy + y^2) + (x^2 + 4xy + 3y^2)$$

$$= 4x^2 - 4xy + y^2 + x^2 + 4xy + 3y^2$$

$$= 5x^2 + 4y^2$$

$$= 5 \times (-4)^2 + 4 \times \left(\frac{3}{2}\right)^2$$

$$= 80 + 9$$

$$= 89$$

$$(2) (x + y)(9x + y) - (3x - y)^2 = (9x^2 + 10xy + y^2) - (9x^2 - 6xy + y^2)$$

$$= 9x^2 + 10xy + y^2 - 9x^2 + 6xy - y^2$$

$$= 16xy$$

$$= 16 \times (-4) \times \frac{3}{2}$$

$$= -96$$

11 [鎌倉学園]

解説

$$x^2 - xy + y^2 = (x^2 + 2xy + y^2) - 3xy$$

$$= (x + y)^2 - 3xy$$

この式に $x + y = 3$, $xy = 1$ を代入すると
 $(x + y)^2 - 3xy = 3^2 - 3 \times 1$
 $= 6$

1

解説

$$(1) \frac{(x - 2)(x - 3)}{4} - \frac{(x - 2)(x - 3) + 4}{5} = \frac{5(x - 2)(x - 3) - 4(x - 2)(x - 3) - 4 \times 4}{20}$$

$$= \frac{(x - 2)(x - 3) - 16}{20}$$

$$= \frac{(x^2 - 5x + 6) - 16}{20}$$

$$= \frac{x^2 - 5x - 10}{20}$$

$$(2) \frac{(1 - x)(3x - 1)}{3} - \frac{(-3x + 2)(x + 1)}{4} = \frac{4(1 - x)(3x - 1) - 3(-3x + 2)(x + 1)}{12}$$

$$= \frac{4(-3x^2 + 4x - 1) - 3(-3x^2 - x + 2)}{12}$$

$$= \frac{-12x^2 + 16x - 4 + 9x^2 + 3x - 6}{12}$$

$$= \frac{-3x^2 + 19x - 10}{12}$$

$$(3) \frac{(a - 2)^2}{2} - \frac{a(a - 1)}{4} + 2a + 1(a - 1) = \frac{2(a - 2)^2 - a(a - 1) + 8(a + 1)(a - 1)}{4}$$

$$= \frac{2(a^2 - 4a + 4) - (a^2 - a) + 8(a^2 - 1)}{4}$$

$$= \frac{2a^2 - 8a + 8 - a^2 + a + 8a^2 - 8}{4}$$

$$= \frac{9a^2 - 7a}{4}$$

$$(4) \left(\frac{x - 1}{2}\right)^2 - \frac{(x - 2)(x + 3)}{4} + \frac{3}{2}x = \frac{(x - 1)^2}{4} - \frac{(x - 2)(x + 3)}{4} + \frac{3}{2}x$$

$$= \frac{(x - 1)^2 - (x - 2)(x + 3) + 3x \times 2}{4}$$

$$= \frac{x^2 - 2x + 1 - x^2 - x + 6 + 6x}{4}$$

$$= \frac{3x + 7}{4}$$

$$(5) \frac{(x - 6y)(x + 2y)}{2} - \frac{(x - 3y)^2}{3} = \frac{3(x - 6y)(x + 2y) - 2(x - 3y)^2}{6}$$

$$= \frac{3(x^2 - 4xy - 12y^2) - 2(x^2 - 6xy + 9y^2)}{6}$$

$$= \frac{3x^2 - 12xy - 36y^2 - 2x^2 + 12xy - 18y^2}{6}$$

$$= \frac{x^2 - 54y^2}{6}$$

$$(6) \frac{(x - 2y)^2}{3} - \frac{x(2x - 3y)}{4} + \frac{(7x - 4y)y}{12}$$

$$= \frac{4(x - 2y)^2 - 3x(2x - 3y) + (7x - 4y)y}{12}$$

$$= \frac{4(x^2 - 4xy + 4y^2) - 3x(2x - 3y) + (7x - 4y)y}{12}$$

$$= \frac{4x^2 - 16xy + 16y^2 - 6x^2 + 9xy + 7xy - 4y^2}{12}$$

$$= \frac{-2x^2 + 12y^2}{12} = \frac{-x^2 + 6y^2}{6}$$

2 [京都産業大]

解説

$$\begin{aligned}(x+y+z)(x-y-z) &= (x+(y+z))(x-(y+z)) \\ &= x^2 - (y+z)^2 \\ &= x^2 - (y^2 + 2yz + z^2) \\ &= x^2 - y^2 - z^2 - 2yz\end{aligned}$$

3

解説

$$\begin{aligned}(1) (x-y+1)(x-y-1) + (x+y-2)(x-y+2) \\ &= ((x-y)+1)((x-y)-1) + (x+(y-2))(x-(y-2)) \\ &= ((x-y)^2 - 1^2) + (x^2 - (y-2)^2) \\ &= x^2 - 2xy + y^2 - 1 + x^2 - y^2 + 4y - 4 \\ &= 2x^2 - 2xy + 4y - 5\end{aligned}$$

$$\begin{aligned}(2) (x+y+z)^2 + (x+y-z)^2 + (y+z-x)^2 + (z+x-y)^2 \\ &= (x^2 + y^2 + z^2 + 2xy + 2yz + 2zx) + (x^2 + y^2 + z^2 + 2xy - 2yz - 2zx) \\ &\quad + (x^2 + y^2 + z^2 - 2xy + 2yz - 2zx) + (x^2 + y^2 + z^2 - 2xy - 2yz + 2zx) \\ &= 4x^2 + 4y^2 + 4z^2\end{aligned}$$

4 [函館大]

解説

$$\begin{aligned}(1) (x+y)(x-y)(x^2+y^2)(x^4+y^4) &= (x^2-y^2)(x^2+y^2)(x^4+y^4) = (x^4-y^4)(x^4+y^4) \\ &= x^8 - y^8 \\ (2) (x-1)^2(x^2+x+1)^2 &= ((x-1)(x^2+x+1))^2 = (x^3-1)^2 = x^6 - 2x^3 + 1 \\ (3) (1+a)(1-a^3+a^6)(1-a+a^3) &= (1+a)(1-a+a^3)(1-a^3+a^6) \\ &= (1+a^3)(1-a^3+a^6) \\ &= 1+a^9\end{aligned}$$

5

解説

$$\begin{aligned}(1) x(x+2)(x+3)(x+5) &= (x(x+5))(x+2)(x+3) \\ &= (x^2+5x)(x^2+5x+6) \\ &= (x^2+5x)(x^2+5x+6) \\ &= (x^2+5x)^2 + 6(x^2+5x) \\ &= x^4 + 10x^3 + 25x^2 + 6x^2 + 30x \\ &= x^4 + 10x^3 + 31x^2 + 30x \\ (2) (x-1)(x+1)(x+2)(x+4) &= ((x-1)(x+4))(x+1)(x+2) \\ &= (x^2+3x-4)(x^2+3x+2) \\ &= ((x^2+3x)-4)((x^2+3x)+2) \\ &= (x^2+3x)^2 - 2(x^2+3x) - 8 \\ &= x^4 + 6x^3 + 9x^2 - 2x^2 - 6x - 8 \\ &= x^4 + 6x^3 + 7x^2 - 6x - 8 \\ (3) (x-1)(x-3)(x+1)(x+3) &= ((x-1)(x+1))(x-3)(x+3) \\ &= (x^2-1)(x^2-9) \\ &= x^4 - 10x^2 + 9 \\ (4) (x+2)(x+3)(x+4)(x+6) &= ((x+2)(x+6))(x+3)(x+4) \\ &= (x^2+8x+12)(x^2+7x+12)\end{aligned}$$

$$\begin{aligned}&= ((x^2+12)+8x)((x^2+12)+7x) \\ &= (x^2+12)^2 + 15x(x^2+12) + 56x^2 \\ &= x^4 + 24x^2 + 144 + 15x^3 + 180x + 56x^2 \\ &= x^4 + 15x^3 + 80x^2 + 180x + 144\end{aligned}$$

6

解説

$$(1) x + \frac{1}{x} = 3 \text{ の両辺を } 2 \text{ 乗すると } \left(x + \frac{1}{x}\right)^2 = 3^2$$

$$\text{よって } x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 9$$

$$\text{すなわち } x^2 + 2 + \frac{1}{x^2} = 9$$

$$\text{したがって } x^2 + \frac{1}{x^2} = 7$$

$$(2) x - \frac{1}{x} = 2 \text{ の両辺を } 2 \text{ 乗すると } \left(x - \frac{1}{x}\right)^2 = 2^2$$

$$\text{よって } x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 4$$

$$\text{すなわち } x^2 - 2 + \frac{1}{x^2} = 4$$

$$\text{したがって } x^2 + \frac{1}{x^2} = 6$$

$$(3) x - \frac{1}{x} = 3 \text{ の両辺を } 2 \text{ 乗すると } \left(x - \frac{1}{x}\right)^2 = 3^2$$

$$\text{よって } x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 9$$

$$\text{すなわち } x^2 + \frac{1}{x^2} = 11$$

$$\begin{aligned}\text{したがって } x^2 - 3x + 2 + \frac{3}{x} + \frac{1}{x^2} &= x^2 + \frac{1}{x^2} - 3\left(x - \frac{1}{x}\right) + 2 \\ &= 11 - 3 \times 3 + 2 = 4\end{aligned}$$

7

解説

$$\begin{aligned}(1) x^5 \text{ の項は } x^3 \cdot 2x^2 + 3x^2 \cdot x^3 &= 5x^5 \\ x^3 \text{ の項は } x^3 \cdot 1 + 3x^2 \cdot (-x) + 2x \cdot 2x^2 + 7 \cdot x^3 &= 9x^3 \\ \text{よって, } x^5 \text{ の係数は } 5, x^3 \text{ の係数は } 9\end{aligned}$$

【参考】 すべて展開すると

$$(\text{与式}) = x^6 + 5x^5 + 7x^4 + 9x^3 + 15x^2 - 5x + 7$$

(2) 展開式において, xyz の項が出てくる計算だけを取り出すと

$$5x \cdot (-2yz) - y \cdot 10zx - 2z \cdot 5xy = -10xyz - 10xyz - 10xyz = -30xyz$$

よって, 求める係数は -30

8

解説

$$\begin{aligned}(1) P &= 2013^2 - 3 \times 2012^2 + 2 \times 2013 \times 2012 + 3 \times 2012 \times 2011 - 3 \times 2011 \times 2013 \\ &= (2012+1)^2 - 3 \times 2012^2 + 2 \times (2012+1) \times 2012 \\ &\quad + 3 \times 2012 \times (2012-1) - 3 \times (2012-1) \times (2012+1) \\ &= (a+1)^2 - 3a^2 + 2(a+1)a + 3a(a-1) - 3(a-1)(a+1) \\ &= a^2 + 2a + 1 - 3a^2 + 2a^2 + 2a + 3a^2 - 3a - 3a^2 + 3\end{aligned}$$

$$= a + 4$$

$$(2) (1) \text{ から } P = 2012 + 4 = 2016$$

9

解説

$$\begin{aligned}(1) 2083^2 - 2081 \times 2085 &= 2083^2 - (2083-2) \times (2083+2) \\ &= 2083^2 - (2083^2 - 2^2) \\ &= 2^2 \\ &= 4\end{aligned}$$

$$\begin{aligned}(2) 5718 \times 5712 - 5715^2 &= (5715+3) \times (5715-3) - 5715^2 \\ &= (5715^2 - 3^2) - 5715^2 \\ &= -3^2 \\ &= -9\end{aligned}$$

$$\begin{aligned}(3) 3916 \times 3912 - 3910 \times 3918 &= (3914+2) \times (3914-2) - (3914-4) \times (3914+4) \\ &= (3914^2 - 2^2) - (3914^2 - 4^2) \\ &= -2^2 + 4^2 \\ &= 12\end{aligned}$$

$$\begin{aligned}(4) 4839 \times 4833 - 4840 \times 4832 &= (4836+3) \times (4836-3) - (4836+4) \times (4836-4) \\ &= (4836^2 - 3^2) - (4836^2 - 4^2) \\ &= -3^2 + 4^2 \\ &= 7\end{aligned}$$

10 [星稜]

解説

大, 中, 小の3つの円において, 斜線部分は大円から小円と中円を除いた図形である。よって, 求める面積は

$$\begin{aligned}\pi \times \left(\frac{1}{2}(x+y)\right)^2 - \pi \times \left(\frac{1}{2}x\right)^2 - \pi \times \left(\frac{1}{2}y\right)^2 &= \frac{1}{4}\pi((x+y)^2 - x^2 - y^2) \\ &= \frac{1}{4}\pi \times 2xy \\ &= \frac{1}{2}\pi xy \text{ (cm}^2\text{)}\end{aligned}$$

11

解説

$$\begin{aligned}T &= \frac{1}{2}\pi b^2 \\ S &= \frac{1}{2}\pi(a+b)^2 - \frac{1}{2}\pi a^2 - \frac{1}{2}\pi b^2 \\ &= \frac{1}{2}\pi(a^2 + 2ab + b^2 - a^2 - b^2) = \pi ab \\ S = 3T \text{ のとき } \pi ab &= \frac{3}{2}\pi b^2 \\ \text{よって } a &= \frac{3}{2}b \quad \text{したがって } a : b = 3 : 2\end{aligned}$$

12 [栃木県]

解説

中央の数を n とすると、連続する5つの整数は小さい方から $n-2, n-1, n, n+1, n+2$ と表すことができる。

よって

$$\begin{aligned}(n+2)(n+1)-(n-2)(n-1) &= n^2+3n+2-(n^2-3n+2) \\ &= n^2+3n+2-n^2+3n-2 \\ &= 6n\end{aligned}$$

よって、最も大きい数と2番目に大きい数の積から、最も小さい数と2番目に小さい数の積をひくと、中央の数の6倍になる。

13 [香川県]

解説

整数 M の十の位の数を x 、一の位の数を y とすると

$$M=10x+y, N=x+y$$

と表すことができる。

$$\begin{aligned}\text{よって } M^2-N^2 &= (10x+y)^2-(x+y)^2 \\ &= 100x^2+20xy+y^2-(x^2+2xy+y^2) \\ &= 99x^2+18xy \\ &= 9(11x^2+2xy)\end{aligned}$$

$11x^2+2xy$ は整数であるから、 M^2-N^2 は9の倍数である。

14 [福岡県]

解説

n を整数として、2番目に小さい数を $2n$ と表すと、最も小さい数は $2n-2$ 、2番目に大きい数は $2n+8$ 、最も大きい数は $2n+10$ と表される。

$$\begin{aligned}\text{よって } & ((2n+10)+2n)^2-((2n+8)+(2n-2))^2 \\ &= (4n+10)^2-(4n+6)^2 \\ &= 16n^2+80n+100-(16n^2+48n+36) \\ &= 32n+64 \\ &= 32(n+2)\end{aligned}$$

$n+2$ は整数であるから、 $32(n+2)$ は32でわりきれぬ。

したがって、4つの偶数において、最も大きい数と2番目に小さい数の和の2乗から、2番目に大きい数と最も小さい数の和の2乗をひいた差は、32でわりきれぬ。

1

解説

- (1) $(a+b-c)^2-(a-b+c)^2$
 $= \{(a+b-c)\}^2-\{(a-b+c)\}^2$
 $= a^2+2a(b-c)+(b-c)^2-\{a^2-2a(b-c)+(b-c)^2\}$
 $= a^2+2a(b-c)+(b-c)^2-a^2+2a(b-c)-(b-c)^2$
 $= 4a(b-c)=4ab-4ac$
- (2) $(x^2+y^2)^2+(x+y)^2(x-y)^2=(x^2+y^2)^2+\{(x+y)(x-y)\}^2$
 $= (x^2+y^2)^2+(x^2-y^2)^2$
 $= x^4+2x^2y^2+y^4+x^4-2x^2y^2+y^4$
 $= 2x^4+2y^4$
- (3) $(a+b-c)(a-b+c)+(b-c)^2=\{(a+b-c)\}(a-(b-c))+\{(b-c)\}^2$
 $= a^2-(b-c)^2+(b-c)^2=a^2$
- (4) $(3x+y-z)^2-(3x+y)(3x+y-2z)$
 $= \{(3x+y)-z\}^2-(3x+y)\{(3x+y)-2z\}$
 $= (3x+y)^2-2(3x+y)z+z^2-(3x+y)^2+2(3x+y)z$
 $= z^2$
- (5) $(x+3)(x-5)(x+5)(x-3)=(x+3)(x-3)\times(x-5)(x+5)$
 $= (x^2-9)(x^2-25)$
 $= x^4-34x^2+225$
- (6) $(x-1)(x+1)(x^2+1)(x^4+1)=(x^2-1)(x^2+1)(x^4+1)$
 $= (x^4-1)(x^4+1)=x^8-1$
- (7) $(2x+1)(x+2)(2x-1)(x-2)=(2x+1)(2x-1)\times(x+2)(x-2)$
 $= (4x^2-1)(x^2-4)=4x^4-17x^2+4$
- (8) $(x^2+xy+y^2)(x^2-xy+y^2)(x^4-x^2y^2+y^4)$
 $= \{(x^2+y^2)+xy\}\{(x^2+y^2)-xy\}(x^4-x^2y^2+y^4)$
 $= \{(x^2+y^2)^2-(xy)^2\}(x^4-x^2y^2+y^4)$
 $= (x^4+2x^2y^2+y^4-x^2y^2)(x^4-x^2y^2+y^4)$
 $= (x^4+x^2y^2+y^4)(x^4-x^2y^2+y^4)$
 $= \{(x^4+y^4)+x^2y^2\}\{(x^4+y^4)-x^2y^2\}$
 $= (x^4+y^4)^2-(x^2y^2)^2=x^8+2x^4y^4+y^8-x^4y^4$
 $= x^8+x^4y^4+y^8$
- (9) $(x-3y)(x+3y)(x^2+3xy+y^2)(x^2-3xy+y^2)$
 $= (x^2-9y^2)\{(x^2+y^2)+3xy\}\{(x^2+y^2)-3xy\}$
 $= (x^2-9y^2)\{(x^2+y^2)^2-(3xy)^2\}$
 $= (x^2-9y^2)(x^4+2x^2y^2+y^4-9x^2y^2)$
 $= (x^2-9y^2)(x^4-7x^2y^2+y^4)$
 $= x^2(x^4-7x^2y^2+y^4)-9y^2(x^4-7x^2y^2+y^4)$
 $= x^6-7x^4y^2+x^2y^4-9x^4y^2+63x^2y^4-9y^6$
 $= x^6-16x^4y^2+64x^2y^4-9y^6$
- (10) $(a+b+c)(-a+b+c)+(a-b+c)(a+b-c)$
 $= -\{a+(b+c)\}\{a-(b+c)\}+\{a-(b-c)\}\{a+(b-c)\}$
 $= -\{a^2-(b+c)^2\}+\{a^2-(b-c)^2\}$
 $= -a^2+(b+c)^2+a^2-(b-c)^2$
 $= (b+c)^2-(b-c)^2$

$$\begin{aligned}&= b^2+2bc+c^2-(b^2-2bc+c^2) \\ &= 4bc\end{aligned}$$

- (11) $(a+b+c)^2-(a-b-c)(a+b-c)-2(b+c)(a+b)$
 $= \{(a+b)+c\}^2-\{(a-c)-b\}\{(a-c)+b\}-2(b+c)(b+a)$
 $= (a+b)^2+2(a+b)c+c^2-\{(a-c)^2-b^2\}-2\{b^2+(c+a)b+ca\}$
 $= a^2+2ab+b^2+2ac+2bc+c^2-(a^2-2ac+c^2-b^2)-2\{b^2+bc+ab+ca\}$
 $= 2ca$
- (12) 与式 $= \{(a+b)^2-1\}\{(a-b)^2-1\}$
 $= (a^2+2ab+b^2-1)(a^2-2ab+b^2-1)$
 $= \{(a^2+b^2-1)+2ab\}\{(a^2+b^2-1)-2ab\}$
 $= (a^2+b^2-1)^2-(2ab)^2$
 $= a^4+b^4+1+2a^2b^2-2b^2-2a^2-4a^2b^2$
 $= a^4+b^4-2a^2b^2-2a^2-2b^2+1$

2 [桐光学園]

解説

$x+y=1$ の両辺を2乗すると

$$\begin{aligned}(x+y)^2 &= 1^2 \\ x^2+2xy+y^2 &= 1 \\ x^2+y^2+2xy &= 1\end{aligned}$$

これに $x^2+y^2=5$ を代入して

$$\begin{aligned}5+2xy &= 1 \\ 2xy &= -4 \\ xy &= -2\end{aligned}$$

$$\begin{aligned}\text{よって } \frac{1}{x}+\frac{1}{y} &= \frac{y+x}{xy} = \frac{x+y}{xy} \\ &= -\frac{1}{2}\end{aligned}$$

3

解説

$$\begin{aligned}(1) (a+b)(c+d) &= ac+ad+bc+bd \\ &= ac+1+1+bd \\ &= ac+bd+2\end{aligned}$$

ここで、 $(a+b)(c+d)=5\times 2$ であるから

$$\begin{aligned}ac+bd+2 &= 10 \\ ac+bd &= 8\end{aligned}$$

(2) $b=5-a, d=2-c$ を $ad=bc$ に代入して

$$\begin{aligned}a(2-c) &= (5-a)c \\ 2a-ac &= 5c-ac \\ 2a &= 5c \\ \frac{a}{c} &= \frac{5}{2}\end{aligned}$$

4 [金沢工業大]

解説

$$x^2+y^2+z^2=(x+y+z)^2-2(xy+yz+zx)=7^2-2\cdot 14=21$$

5 [東北学院大]

解説

$$\begin{aligned} \frac{1}{1-a} + \frac{1}{1+a} &= \frac{1+a+1-a}{(1-a)(1+a)} = \frac{2}{1-a^2}, \\ \frac{2}{1-a^2} + \frac{2}{1+a^2} &= 2 \cdot \frac{1+a^2+1-a^2}{(1-a^2)(1+a^2)} = \frac{4}{1-a^4}, \\ \frac{4}{1-a^4} + \frac{4}{1+a^4} &= 4 \cdot \frac{1+a^4+1-a^4}{(1-a^4)(1+a^4)} = \frac{8}{1-a^8} \end{aligned}$$

よって (与式) $= \frac{8}{1-a^8} + \frac{8}{1+a^8}$

$$= 8 \cdot \frac{1+a^8+1-a^8}{(1-a^8)(1+a^8)} = \frac{16}{1-a^{16}}$$

6

解説

小さい方の整数を5でわったときの商を n (n は0以上の整数) とすると、

$$\text{小さい方の整数は } 5 \times n + 2 = 5n + 2$$

$$\text{大きい方の整数は } 5n + 3$$

と表される。

この2つの整数の積は

$$\begin{aligned} (5n+2)(5n+3) &= 25n^2 + 25n + 6 \\ &= 25n^2 + 25n + 5 + 1 \\ &= 5(5n^2 + 5n + 1) + 1 \end{aligned}$$

$5n^2 + 5n + 1$ は整数であるから、5でわったときの余りは1である。

7 [大分県]

解説

(1) 例えば、2つの自然数を1と3とすると

$$A = 1 + 3 = 4$$

$$B = 3 - 1 = 2$$

$$\text{このとき } X = 4^2 - 2^2 = 12$$

X は8の倍数ではない。

よって (ア) 1 (イ) 3 (ウ) 12

(2) $A = a + b$, $B = a - b$ である。

$$\begin{aligned} \text{このとき } X &= (a+b)^2 - (a-b)^2 \\ &= a^2 + 2ab + b^2 - (a^2 - 2ab + b^2) \\ &= 4ab \end{aligned}$$

a , b は自然数であるから、 $4ab$ は4の倍数である。

したがって、 X はいつでも4の倍数である。

(3) (2) より $4ab = 2016$

$$ab = 504$$

この式を満たす a , b のうち、 $a - b$ の値がもっとも小さくなるものをみつける。

$504 = 2^3 \times 3^2 \times 7$ であるから、求める2つの自然数は

$$21, 24$$